Chen, W. et al. (Eds.) (2016). Proceedings of the 24th International Conference on Computers in Education. India: Asia-Pacific Society for Computers in Education

Geometry-via-Gestures: Design of a gesture based application to teach 3D Geometry

Lakshmi T.G. [a*], Soumya NARAYANA[a], Prajish PRASAD[a], Sahana MURTHY[a] & Sanjay CHANDRASEKHARAN[b]

[a]*Inter-disciplinary Program in Educational Technology, IIT Bombay, India*

[b] *Homi Bhabha Centre for Science Education, Tata Institute of Fundamental Research, India*

*tglakshmi@iitb.ac.in

Abstract: Geometry holds a special place in Mathematics. Learning of Geometry requires understanding and integrating a wide range of topics such as coordinates, shapes, theorems, proofs, properties and formulae, to name a few. In 3D geometry, students find difficult the manipulation of 3D objects and interpreting their structure and properties, such as volume and surface area. Research shows that students mainly focus on the formulae and the numerical operations to calculate the volume or the surface area of a 3D object, and not on visualizing the 3D objects. Various media such as images, animation, simulations and 3D geometric software have been used to help learners visualize 3D shapes. However, the affordances provided by these media do not fully support the construction and manipulation of 3D objects. Based on an embodied interaction approach, we have designed a gesture-based application for high school students to learn the properties of 3D objects. In this paper, we describe the design of the application, "Geometry-via-Gestures" (GvG), which enables learners to construct a right circular cylinder using gestures and derive its volume. We present the results of the first prototype pilot and the proposed redesign.

Keywords: 3D geometry, gestures, embodied cognition, 3D property

1. Introduction

Geometry has always been an essential part of the mathematics curriculum. Concepts in geometry are used across various domains such as computer graphics, medical imaging, image processing, robotics etc. Bloom (Bloom, 1998) stresses the importance of geometry "not only in its role in day-to-day activities but also as a vehicle to promote visualization in mathematics and many of the sciences". Practice in many science disciplines require building three-dimensional geometrical models, which are visualized and manipulated. Howe and Yasu(Howe & Yasu, 1989) state that "most basic science concepts are related in some way to a three-dimensional object or model whose understanding requires an act of imagination in the formation of a mental image."

Since the learning of 3D geometry involves visualizing and manipulating various 3D structures, and visualizing such 3D structures is difficult, teaching and learning of 3D geometry is a challenge faced by many. A common lament by teachers of geometry is that learners merely memorize formulas of various properties of 3D structures, like volume and surface area, without understanding how they have been derived. Research findings also show that students mainly focus on the formulae and the numerical operations required in calculating the volume or the surface area of a solid, and completely ignore the structure of the unit measures (Pittalis & Constantinos, 2010).

For learners to conceptually understand the idea of volume or surface area, they need to form connections between the formula and the structure of the solid. Marios Pittalis et al, (Pittalis & Constantinos, 2010) in their work on geometrical thinking, express the importance of constructing 3D objects from constituent 2D primitives. According to their observation, understanding the properties of a solid is equivalent to understanding the characteristic parts of 3D shapes. They claim that 3D geometry thinking is closely connected to students' ability to calculate properties of 3D objects, such as volume and surface area. To enable learners to understand various properties of 3D structures, it is important to help them understand how 3D structures are generated from their constituent 2D primitives. Traditionally, physical models and computer applications have been used to teach 3D geometry. However, physical models do not lend themselves to manipulations such as breaking open, changing shape and 2D - 3D translation, to name a few. Although the learner is able to interact with the physical model, the learner cannot construct intermediate structures while translating from 2D to 3D. Animations and simulations do provide a certain degree of interaction. However, they are restricted by the affordances provided by the keyboard and the mouse. Gestures offer a more natural form of human and computer interaction. The idea that knowledge is embodied and expressed through bodies (Goldin-Meadow & Beilock, 2010) suggest gestures could be a natural form of interaction for learning 3d geometry. With the advancement of technology, it is now possible to use gestures for intuitive and natural user interactions. In this paper, we describe the initial design of our application "Geometry-via-Gestures" (GvG). GvG, along with orchestration of activities, enables learners to visualize and manipulate 3D structures using gestures. This paper reports evidence of improvement in the learning process while using the application. We also identify additional features which could strengthen the pedagogical design of GvG, based on learner interaction with the application and interviews. We conclude with a proposed redesign and recommendations.

2. Theoretical Underpinnings

"Embodiment is the property of being manifest in and as a part of the world" (Dourish, 2004). Philosophers and psychologists have taken the position that the body plays a significant role in cognition. There have been different views of the role of body in cognition, starting from individualistic aspect such as reasoning (O'Donovan-Anderson et al., 2000), to a larger sociocultural world (Hutchins, 2014). Cognition, as per these theorists, derives from, and repurposes, action. Such theories seek unification of body, action and mind. Embodiment theories treat cognition as participation. The bodily participation then becomes a part of the conceptual structure.

Technology offers different levels of embodiment. The desktop technology is very much static, whereas new media offers a more dynamic medium (Victor, 2014). New media offers multi-touch, sensor and tangible interfaces that can be utilized to exploit more bodily interactions. The new media interfaces are not limited to interactions, but can be extended to experiences as well (Farr et al., 2012). There are two general ways in which the embodiment literature is related to technology. The first is where current interaction technology is considered wanting, and embodiment is considered as a way of developing new interaction technology. This stance of embodied interaction aims at creating new media, which creates new representations and interactions, which can become new thinking tools. In this theme, education technology (ET) researchers focus on creating new conceptual structures for education. The lack of educational theory to inform or analyze such learning through new media limits development of such systems (Abrahamson & Sánchez-García, 2016).

In the second way in which embodiment is related to technology, specific design decisions are supported by appealing to embodiment as a design principle. With the recent advancement in technologies, embodiment is now considered as an interaction feature. Natural User Interfaces (NUI) allow computer interactions through user's body movements. Some examples of NUI comes from gaming environments where controllers such as Nintendo Wii Remote, Wii fit, Microsoft Kinect etc. allow gamers to engage directly with virtual environments through their body. In ET, this design approach could include an appeal to the compatibility between the embodied nature of the concept/skill (say mental rotation) and the embodied technology to teach it (say actual rotation of complex structures). Learning gains have been reported using this stance of embodiment (Hung et al., 2014). A related theme could be an appeal to embodiment as a user-friendly feature in general, which could make concept-learning interesting or motivating. This paper takes the above stance, as learning 3D geometry is inherently an embodied process, as teachers and learners talk about 3D objects through gestures.

When the process of action is included in perception and memory making the self is associated with these processes, and this leads to better cognition (Hung et al., 2014). Actions such as physical manipulation and imagined manipulation can lead to enhanced comprehension (Alibali et al., 1999). There is empirical evidence that the actions performed by a learner in the "field of experience" positively influences internal representations (Novak et al., 2014). Gestures are special kind of actions that people produce when talking (McNeill, 2012). They are representational modes that convey spatial, relational and embodied concepts (Chue, 2015). Gestures contain information that supplements speech, or they can be independent (Goldin-Meadow, 2006). Gestures are particularly good at expressing spatial and motor information (Alibali, 2005). For example, while giving directions such as "Turn right/ turn left" we also tend to move our hands in the corresponding direction. This movement conveys the spatial information as well as the mental image present in the speaker's mind. Information about the problem solving process and problem representation is available in gestures (Hung et al., 2014). Gestures have been used as a problem solving strategy in mathematics with higher learning gains (Cook & Goldin-Meadow, 2006). Additionally, Cook & Goldin-Meadow (Cook & Goldin-Meadow, 2006), claim that adding gestures to instruction promotes learning. In the teaching of astronomy to school students gestures have been found to be an appropriate pedagogy, making concrete models dynamic (Padalkar & Ramadas, 2011).

Gestures are classified into deictic, metaphoric, beat and iconic (McNeill, 1992). Deictic, metaphoric and iconic are categorized under representational form as they convey semantics, whereas beat gesture is used more for emphasis and does not convey any meaning. We speculate that in our application Metaphoric gestures will be used by the learner, but analysing the learner's gestures for the type is beyond the scope of this paper.

3. Pedagogical Design of GvG

3D geometry understanding includes spatial structuring, conceptualization of mathematical properties, and conceptualization of measurement. Spatial structuring of object involves identifying spatial components, combining them into spatial composites, establishing interrelations among components and composites. Literature emphasizes the need to conceptually link 2D and 3D objects and also derive the properties of 3D objects. Representing 3D objects by 2D figures is a 3D geometry ability (Pittalis & Constantinos, 2010). The geometry knowledge of 3D structures is built on prior geometry knowledge of 2D primitives. The traces that form during the process of building 3D structure from 2D structure are attention anchors that aid students to relate 2D primitives with 3D structure formation (Abrahamson & Sánchez-García, 2016). The process of construction of a 3D object from its constituent 2D shapes will help learners derive the properties of the solid they make. Understanding properties of solid means (Pittalis & Constantinos, 2010):

- understanding characteristic parts of 3D shapes, comparative relations between same or different structural parts, how elements of a solid are interrelated
- conceptualization of numerical operations and links of formulas with structure of solid
- understanding visualization of internal structure of solid which contributes to understanding of how volume is calculated

One of the modes of learning 3D geometry by experience is to use physical models. In a traditional setup, the teacher brings a 3D artefact to the class and draws the different views on the board. Students do not interact with the 3D objects. For the development of the abstract and concrete aspects of geometry, emphasis has been on experience, rather than theory, which is found lacking in the traditional setup (Hansen et al., 1998).

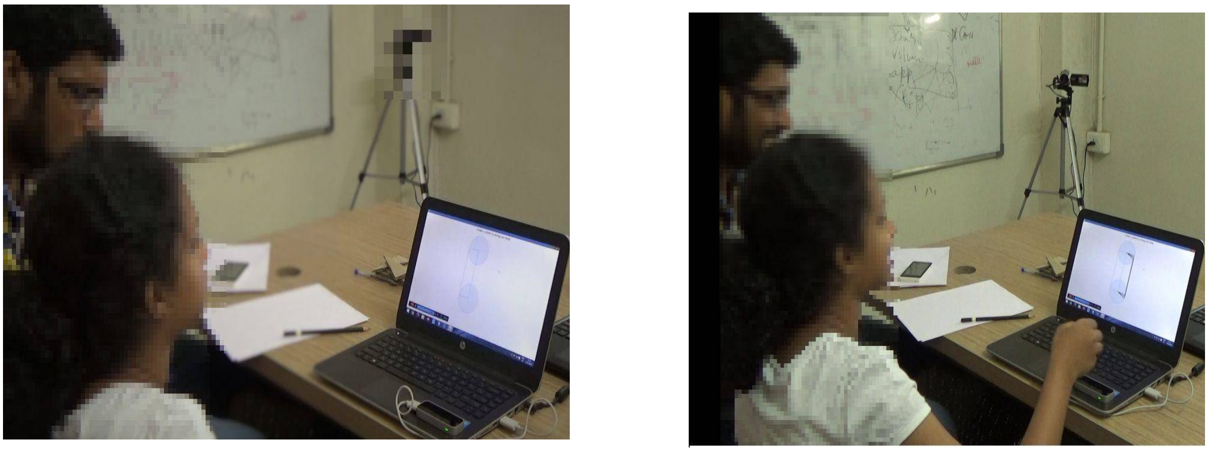
We have designed three learner tasks with the application based on the embodied cognition, gesture and 3D geometry literature for the three learning objectives. Each activity maps to a specific learning objective and the learner interacts with the application to achieve the objective. Table 1 contains the mapping of the learning objectives, application features and literature support. The researcher/mediator is with the learner during the task, prodding, guiding, hinting and collating learner insights during the interaction.

Table 1: Mapping Learning Objective, Features of application and Theory.

Sno	Learning Objective	Feature	Theoretical Backing
1	Learner should be able to use gestures to move on screen	Task 1	Run-in Period (Hung et al.,2014)

	3D object in various orientations.		
2	Learner should be able to explain the formation of a 3D object (right circular cylinder) from a known basic 2D structures (rectangle, circle)	Task 2	Representing 3D objects by 2D figures (Battista, 2007; Pittalis and Constantinos, 2010)
3	Learner should be able to conceptually derive the volume of a 3D object (right circular cylinder)	Task 3	Conceptualizing links of formulae with structure of solid (Battista, 2007; Pittalis and Constantinos, 2010)

In the application, the task of formation of a right circular cylinder (task 2) is done using gestures. The student will see a rectangle with two circles - one at the top and one at the bottom. The two sides of the rectangle forms the radius of the two circles (see Figure 1a.). The student uses the sliding gesture in front of the leap motion controller. When the sliding gesture is sensed by the controller, it will be verified by the application code if the gesture has been associated with any action with the current model on-screen. If specific action is associated, as in this case, then the rectangles can be made to move like a 3D cycle wheel (see Figure 1b). This will enable the student to associate the 2D rectangle primitive with the 3D right circular cylinder. Figure 1a-b shows students interacting with the application for this particular task.



a. b.

Figure 1. Student participants interacting with GvG for creating a right circular cylinder using gestures.

4. Study Design

The broad research questions guiding the study were:

1. How do learners use GvG to learn 3D geometry?
2. What features are required to strengthen the pedagogical design of GVG?

Participants: Geometry as a subject is taught in India over several grades, beginning from primary (4th grade) school, making it difficult to pinpoint the optimum age group. Hence we conducted the study on learners in the 6th, 9th and 10th grade. The learners for the GvG application were students residing around the research centre.

Procedure: The study was conducted in an experiment room, by an interviewer, and two observers. A laptop with the GvG application loaded on it was placed before the learner, along with the leap motion controller. Sufficient writing material was also placed close to the student for use if necessary. The whole study was recorded using two video cameras and an audio recorder. The learners were interviewed individually, and the study was spread over two days.

Gestures are the application input interface in GvG. As the learners use gestures to interact, questions were posed by the facilitator. The facilitator had three sets of pre-determined questions to pose before, during and after the activity. The questions posed before the activity aimed at eliciting prior knowledge and identifying misconceptions. During the activity the facilitator asked questions to extract the student's thought process and knowledge construction. The answers by the learners formed the intermediate outcomes. The goal of the post-activity questions was to evaluate knowledge transfer.

Intermediate Evaluation: Questions posed by the facilitator during the activity was expected to make students reflect on what action they did, why they did that particular action/gesture, what happened to the object on screen, what insight did the student gain from this activity, and finally, whether the student could conceptually describe the volume property of the 3D object in terms of the 2D objects. The student answers were evaluated based on a rubric. Two Rubrics with three levels (Expected, Intermediate and Poor) was developed to evaluate responses for questions posed for Task 2 and Task 3. The first rubric was meant to rate the response to the question on the students' observation of transformation to onscreen object, on successful manipulation using gestures. This was applicable for both Task 2 and Task 3 as in both these tasks the students constructed a 3D shape (right circular cylinder) using rectangle (task 1) and using circles (task 2). The second rubric was to rate the response to the question on derivation of volume of the resultant 3D object displayed on screen. This was applicable for Task 3 as the students could explain volume of the right circular cylinder as area of circle multiplied by the height of the circles.

Post-test: In the Post-test a rectangular sheet of cardboard was given to students and they were asked questions on the number of ways in which a right circular cylinder could be formed, using the sheet. A triangular cut-out in a cardboard was provided to students and they were asked to generate 3 dimensional objects using the triangular cut-out. The expected outcome is a triangular prism structure obtained such triangular cut-outs are stacked. Also expected is the conceptual understanding of volume of such a structure. An interview after the Post-test was conducted to understand the role of gesture in the 3D structure formation, usefulness of the tool and additional features they would have preferred. Figure 2 depicts the study steps and the outcome from each of the steps. From the outcomes available the focus of the paper is only on "Learning Processes" and "Application Features".

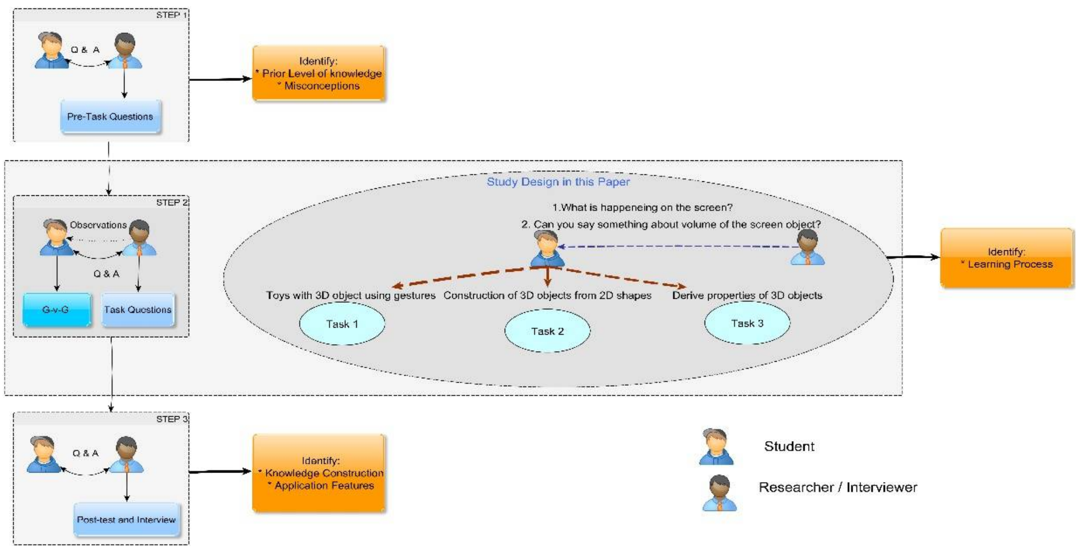


Figure 2. Pilot Study Steps

5. Results

We conducted the pilot with two 6th grade learners, and one each from the 9th and 10th grade. To answer the research questions posed in this paper, we focus on the video recordings and interview transcripts of the 9th (X) and 10th (Y) grader.

5.1 Clinical Interview - Student X

Student X used gestures spontaneously while describing abstract concepts. X used gestures to show a solid object in space while explaining about the concept of volume. To explain area, X used gestures to indicate flatness. This indicates the naturalness of gestures in describing geometrical concepts. X showed signs of giving up during tasks 2 and 3 when the required gesture could not be identified. The facilitator showed the gesture and X performed the same. Confidence started building up when the gesture used by X started producing output on the screen. This observation gives us inputs for the features to be incorporated in our design, namely scaffolds and reflection prompts.

While interacting with the object on screen using gestures, in the rotation of rectangle to form cylinder activity, X accidently discovered the gesture. This caused the rectangles to replicate. Seeing the output, X was excited and tried the gesture more confidently. In task 2, X understood that the rectangle has to follow the circular path. Accordingly the gesture indicated the movement of the rectangle along with the trajectory it has to take. But X was not able to visualize which side of the rectangle is fixed and which side is moving. So learning using this task seems complicated. This seems to indicate the need for scaffolding in the form of pointers and reflective questions.

Before the intervention, x calculated the concepts of area and volume based on the formula. But using the intervention, X could figure out that 3D shapes are composed of 2D shapes. The intervention helped in the learner deriving the formula of the volume from the area and helped in connecting disparate concepts of area and volume. In the post-test, X is able to transfer the learning that volume is the product of area and height, to a new 3D object - Triangular prism.

5.2 Clinical Interview - Student Y

Y used natural gestures extensively even in his general conversations, mostly to emphasize his point. He also used analogies and artefacts during explanations or while conveying ideas. Y would use any available artefact to put forth his idea. Y was positive about technology and could use the leap motion controller in a comfortable manner.

In task 2 Y got the gesture right and successfully understood the concept. Y went a step further and came up with an innovative analogy for the activity -- that of a compass movement. The implication seems to be that the activity helps the student understand the concept of 3D as a composite of 2D shapes. In task 3 Y used gestures which were a variation of the vertical movement. This seems to indicate a desire to move the 2D disk in the vertical direction to form a cylinder, thereby suggesting the intuitiveness of the designated gesture.

Y had a strong misconception regarding area and volume. This prevented him from obtaining the right formula in spite of deducing the volume of cylinder conceptually. This points towards the need for scaffolding, in the form of reflective questions and hints. Y derived the volume of cylinder as ‘Area of individual unit x n divisions’. There was no clarity between ‘n’ and height. This suggests misconceptions overshadow any learning gained by the activity. Due to the misconceptions the volume derivation process could not be extracted from Y.

5.3 Student Learning

In this section we present the answer to the first research question that we posed in the start of section 6. The next section, section 8 answers the second research question in detail. Based on the analysis of student interactions with GvG and interviews, we have come up with the following insights:

1. Participants use gestures intuitively to explain geometric concepts: We noticed several instances where the participants used their hands and gestures to explain various

geometrical concepts. When asked the definition of volume, Student X used fingers to indicate the volume is the space enclosed by an object. Student Y used hands to explain x, y and z coordinates, by placing hands perpendicular to each other. These instances further support our claim that gestures are indeed a natural way to learn and describe geometry concepts.

2. After using GvG, learners are able to derive a 3D shape from its constituent 2D shapes: Prior to interacting with GvG, Student X differentiated 2D and 3D structures by mentioning the "lack of depth". Student Y differentiated 2D and 3D structures by mentioning that there is a "difference in dimensions". After interacting with GvG, students stated that a cylinder is formed by "layers of circles" and "infinite 2D circles placed line by line". One of the participants, when asked about difference between circle and cylinder in task 2, responded: "There are more layers of it. Circle fits to the base, and adding more and more of the same thing". This shows that GvG enabled students to visualize the 3D shape from its 2D primitives.
3. After using GvG, participants are able to derive the volume of a 3D shape using properties of the constituent 2D shapes - Participants were able to use the above information and derive the volume of the cylinder conceptually. Student X was able to conceptually link the concepts of area and volume. Although Student Y had a misconception about the structure of the cylinder, Y was able to determine that length, breadth and height are required to derive the volume of the cylinder and some property of the circle is also needed.

Based on the above insights we conclude that learners are able to derive the properties of 3D shapes such as volume using GvG. These properties can be derived by first construction of the 3D shape from a known 2D shape. The construction of the 3D shape in this application is aided by gestures. The learners are already aware of the 2D shape properties such as area. This knowledge is connected to the activity performed to derive the volume of 3D shapes. During the construction of 3D shapes appropriate traces are visible, which shows the construction happening. These visualizations along with the activity using gesture helps them to connect prior knowledge to derive the formula of volume.

6. Re-Design

6.1 Limitations of GvG

Based on interviews and observations of students using our application, we have identified the following limitations:

- In its present form, the application cannot help override misconceptions. Student Y's misconception of the concepts of volume, area and surface area, proved unsurmountable using the tool in its present form. Reflection questions could help overcome misconceptions.
- Student X's inability to find the appropriate gesture could have easily lead to frustration but for the presence of the facilitator. Scaffolds are therefore needed to prevent loss of interest due to frustration.
- Self-learning mode requires a number of additional scaffolds. In the present form, the facilitator asked several thought-provoking questions to accompany the activity. If such reflection questions are absent, students may not gain sufficient insights into 3D geometry structure.
- If derivation of 3D objects are done from other 3D objects and not from 2D objects as intended, then the link between 2D and 3D geometric structures would not be properly established.

To overcome the above limitations we propose the induction of features proposed from 8.2 – 8.6 in the GvG application.

6.2 Re -Design of Tasks

Geometric solids can be considered as whole, and according to its components. From our student observations, we noticed that students have vague and inconsistent prior knowledge of how solids are formed. Hence, in the redesign, we want to introduce activities from 1 dimension geometry, and gradually progress to 3 dimensional geometry. This would help our tasks prime students to think of a solid in higher dimension in terms of the constituent parts in the lower dimension. Specifically, the tool will have:

1. Task 1 - 1D to 2D transitions and back; may include: line to rectangle, line to circle, line to triangle
2. Task 2 - 2D to 2D transition and back; may include: triangle to square, triangle + square to pentagon, square + square to rectangle etc.
3. Task 3 - 2D to 3D transition and back; may include: rectangle + circle to cylinder, stack of circles to cylinder, stack of rectangles to cuboid, stack of squares to cube etc.
4. Task 4 - 3D to 3D transition and back; may include: stack of cubes to cuboid

6.3 Re-Design for Multiple Representations

Concepts of form, properties and transformations are embedded in the representation of shape. Hence geometrical figures, especially 3D figures, are "rich cognitive structures"(Freudenthal, 1991). Each property of the shape has a corresponding algebraic representation as well. For example, the volume of a cylinder can be visually represented as the space enclosed by the cylinder. The volume of the cylinder can be represented algebraically by the formula pi*r^2*h. Students should be able to understand the relationship between these two representations, and how changing the visualisation affects the algebraic representation and vice-versa. Hence, we intend to ask the student for the constituent algebraic representation for each visual representation at each dimension. Further visual changes in the solid will result in changes in the algebraic formula and vice-versa.

6.4 Scaffolding

Based on students' observation and to facilitate self-learning, we wish to show three levels of scaffolds:

- Level 1: Show right gesture only, in the form of an animated hand movement.
- Level 2: Show expected output only, in the form of a partly completed output.
- Level 3: Show gesture and expected output as animation simultaneously.

Having such scaffolding in each activity will help prevent the student from getting frustrated or losing interest. These levels could pop-up either on demand or after the user has tried out multiple types of gesture, or if the student has tried only one gesture and has not interacted with the system for more than 5 minutes. To equip the application to respond in terms of the latter two possibilities, some form of activity logging will be required, where the leap motion will be used to sense all types of user gestures and log them.

These scaffolds could address the issue of flagging interest, on not being able to identify the correct gesture and also not knowing what the correct outcome would look like. When the facilitator interacted with the student and asked what gesture they were trying to do, why they feel that the gesture is appropriate and what do they expect the outcome to look like, it was clear that the student's idea and thought process was in tune with what was designed. In the stacking of disk activity, the student rightly conjectured that the cylinder could be formed if the circle was lifted.

However, the student was trying the lifting gesture (vertical movement) with all fingers pinched together. The correct gesture was flat palm moved vertically. Conceptually the two gestures are close. However, due to small variation, i.e. the pinched fingers versus open palm, the response was not seen. Here, if the scaffold pops up with an animation of the gesture, the student could imitate it and still reap the benefit of the understanding via gestures.

6.5 Reflective Questions

During the task 3, the student X accidently stumbled upon the gesture (right to left swipe) and saw the output. The output subsequently disappeared when the leap motion controller detected the left to right swipe, which was coded to indicate clear or reset. Thereafter, her focus was completely on using the gesture to complete the task rather than on understanding the concept underlying it. This could adversely affect the efficacy of the intervention. To counter such scenarios, a second type of scaffolding involving reflection questions would be desirable. Ideally, a facilitator would elicit responses from the student by questioning her thought process behind the nature of gesture and the corresponding output. However, in a self-learning mode, a virtual agent with popup questions could play the role of a facilitator and put forth reflection questions that force the student to think along the lines of cause and effect of gesture to the output visualized onscreen. This will in turn help establish the link between 2D and 3D structures. These popup questions could be enabled after the student has interacted with the system for around three minutes.

7. Discussion and Conclusion

Using a gesture based technology, the GVG application has brought in more natural learner interactions with 3D shapes. However, the learner interaction needs to be channelled via meaningful activity. The natural interactions during meaningful activity, combined with the visualizations on screen, helps the learner connect the 2D shape's properties to the 3D shapes.

In this paper, we have tried to extract the process while interacting with the GvG application, beginning with the construction of a 3D shape from a known 2D shape via gestures. The use of the gestures and the visualizations shown during 3D shape construction enable learners to come up with strategies to derive the properties of 3D shapes such as volume. The learners can now extend this learning to other 3D objects and follow the process to derive the properties.

In our study, it is unclear whether gestures function as epistemic actions (Kirsh & Maglio, 1994) that contribute to learning of the concept. We observed that when students were asked to explain concepts like volume, surface area and explain different shapes, they naturally used their hands and gestures to articulate the concept. Hence gestures themselves act as an external representation, primarily to offload the concept. Gestures also act as a complement to language, since students find it more natural to use gestures to explain these concepts. We are not claiming that identification of gesture is contributing to learning. Rather it is the linking of the gesture with the output, and the reflection on what is exactly happening, that contributes to learning. However, this linking of the gesture with the output alone may not contribute to learning. For such a relation to be established, we would require a control experiment.

The visualisations offered by the tool themselves act as an external representation. They are persistent, and act as anchors, allowing students to build solids of higher dimensions by interacting with the tool. Hence while interacting with the tool, there is an interplay between the visualisation, the gesture, and the multiple forms of representation. Which representations predominantly contribute to learning is an open question.

The application of embodiment theory of cognition in the creation of interaction environments is termed "embodied design" by Abrahamson (Abrahamson, 2009). There exists three areas of challenges in embodied design: (i) activities, (ii) materials, (iii) facilitation (Abrahamson & Lindgren, 2014). Activities in embodied design should start from the simplest task which requires only prior knowledge to be utilized in the new environment. The orchestration of learning materials needs to be supported with feedback. Additionally, as the complexity of task increases the tools should support novel actions, which then leads to new conceptual structures being built. Just in time feedback and hints need to be incorporated with the design which will lead the learners to the objective. The redesign features for GvG 2.0 incorporate almost all of the recommendations of embodied design.

In GvG 2.0, with the additional features, the most interesting question will be to find out the individual contribution of gesture and visualization to the learning. Additionally a control group experiment would be able to measure the learning gain using GvG 2.0.

References

Abrahamson, D. (2009). Embodied design: constructing means for constructing meaning. *Educational Studies in Mathematics*, 70(1), 27-47.

Abrahamson, D., & Lindgren, R. (2014). Embodiment and Embodied Design1.

Abrahamson, D., & Sánchez-García, R. (2016). Learning is moving in new ways: The ecological dynamics of mathematics education. *Journal of the Learning Sciences*, *25*(2), 203-239.

Alibali, M. W. (2005). Gesture in spatial cognition: Expressing, communicating, and thinking about spatial information. *Spatial Cognition and Computation*, *5*(4), 307-331.

Alibali, M. W., Bassok, M., Solomon, K. O., Syc, S. E., & Goldin-Meadow, S. (1999). Illuminating mental representations through speech and gesture. *Psychological Science*, *10*(4), 327-333.

Bloom, W. (1988).Spatial and visual aspects *Proceedings of the Sixth International Congress on Mathematical Education* Hungary.

Chue, S., Lee, Y. J., & Tan, K. C. D. (2015). Iconic gestures as undervalued representations during science teaching. *Cogent Education*, *2*(1), 1021554.

Cook, S. W., & Goldin-Meadow, S. (2006). The role of gesture in learning: Do children use their hands to change their minds?. *Journal of cognition and development*, 7(2), 211-232.

Dourish, P. (2004). *Where the action is: the foundations of embodied interaction*. MIT press.

Farr, W., Price, S., & Jewitt, C. (2012). An introduction to embodiment and digital technology research: Interdisciplinary themes and perspectives.

Freudenthal, H. (1991). Revisiting mathematics education Dordrecht.

Goldin-Meadow, S. (2006). Nonverbal communication: The hand's role in talking and thinking. *Handbook of child psychology*.

Goldin-Meadow, S., & Beilock, S. L. (2010). Action’s influence on thought: The case of gesture. *Perspectives on Psychological Science*, 5(6), 664-674.

Hansen, V. L., Malkevitch, J., & Douady, A. (1998). Geometry: Past and Future. In *Perspectives on the Teaching of Geometry for the 21st Century* (pp. 9-28). Springer Netherlands.

Hung, I. C., Lin, L. I., Fang, W. C., & Chen, N. S. (2014). Learning with the body: An embodiment-based learning strategy enhances performance of comprehending fundamental optics. *Interacting with Computers*, *26*(4), 360-371.

Hutchins, E. (2014). The cultural ecosystem of human cognition. *Philosophical Psychology*, 27(1), 34-49.

Howe, A. C., & Vasu, E. S. (1989). The role of language in children's formation and retention of mental images. *Journal of Research in Science Teaching*, *26*(1), 15-24.

Kirsh, D., & Maglio, P. (1994). On distinguishing epistemic from pragmatic action. *Cognitive science*, 18(4), 513-549.

Lakoff, G., & Johnson, M. (2000). Philosophy in the Flesh: The Embodied Mind and Its Challenge to Western Thought.

McNeill, D. (1992). *Hand and mind: What gestures reveal about thought*. University of Chicago press.

McNeil, N. M., & Fyfe, E. R. (2012). “Concreteness fading” promotes transfer of mathematical knowledge. *Learning and Instruction*, *22*(6), 440-448.

Novack, M. A., Congdon, E. L., Hemani-Lopez, N., & Goldin-Meadow, S. (2014). From action to abstraction using the hands to learn math. *Psychological Science*, *25*(4), 903-910.

O'Donovan-Anderson, M., Lakoff, G., & Johnson, M. (2000). Philosophy in the Flesh: The Embodied Mind and Its Challenge to Western Thought.

Padalkar, S., & Ramadas, J. (2011). Designed and spontaneous gestures in elementary astronomy education. *International Journal of Science Education*, 33(12), 1703-1739.

Victor, B. (2014, October). Humane representation of thought: a trail map for the 21st century. In *Proceedings of the 27th annual ACM symposium on User interface software and technology* (pp. 699-699). ACM.