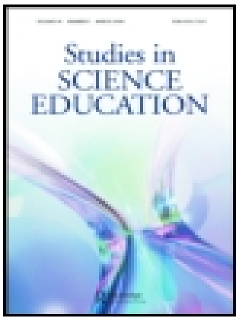


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# Representational competence: towards a distributed and embodied cognition account

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**ABSTRACT**

Multiple external representations (MERs) are central to the practice and learning of science, mathematics and engineering, as the phenomena and entities investigated and controlled in these domains are often not available for perception and action. MERs therefore play a twofold constitutive role in reasoning in these domains. Firstly, MERs stand in for the phenomena and entities that are imagined, and thus make possible scientific investigations. Secondly, related to the above, sensorimotor and imagination-based interactions with the MERs make possible focused cognitive operations involving these phenomena and entities, such as mental rotation and analogical transformations. These two constitutive roles suggest that acquiring expertise in science, mathematics and engineering requires developing the ability to transform and integrate the MERs in that field, in tandem with running operations in imagination on the phenomena and entities the MERs stand for. This core ability to integrate external and internal representations and operations on them – termed representational competence (RC) – is therefore critical to learning in science, mathematics and engineering. However, no general account of this core process is currently available. We argue that, given the above two constitutive roles played by MERs, a theoretical account of representational competence requires an explicit model of how the cognitive system interacts with external representations, and how imagination abilities develop through this process. At the applied level, this account is required to develop design guidelines for new media interventions for learning science and mathematics, particularly emerging ones that are based on embodied interactions. As a first step to developing such a theoretical account, we review the literature on learning with MERs, as well as acquiring RC, in chemistry, biology, physics, mathematics and engineering, from two perspectives. First, we focus on the important theoretical accounts and related empirical studies, and examine what is common about them. Second, we summarise the major trends in each discipline, and then bring together these trends. The results show that most models and empirical studies of RC are framed within the classical information processing approach, and do not take a constitutive view of external representations. To develop an account compatible with the constitutive view of external representations, we outline an interaction-based theoretical account of RC, extending recent advances in distributed and embodied cognition.

**KEYWORDS**
Multiple external representations (MER); representational competence (RC); distributed cognition; embodied cognition; MER integration; new media

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## 1. Introduction

Modern science deals with entities and phenomena that cannot be directly perceived or acted on, because they are too small (atoms, DNA, cells, etc.), too big (galaxies, stars, tectonic plates, etc.) happen in timescales that are difficult to perceive (milliseconds – chemical reactions, millennia – evolution), and are complex (feedback loops between levels and timescales). Understanding and analysing these imperceptible and complex entities and phenomena thus involve imagining and modelling them in detail, and developing indirect measures and novel external representations (symbolic elements that stand in for the actual entities and phenomena) that help in this imagination process. Multiple external representations (MERs) are thus embedded in science practice, and play a constitutive role in developing new models, drawing inferences, making predictions, supporting claims and developing consensus. Ideas and information in science are distributed across MERs (Johnstone, 1991; Lesh, Post, & Behr, 1987; Tsui & Treagust, 2013), and learning and practising science is impossible without gaining expertise in interacting with MERs, imagining with them, and learning to generate them.

Imagined mental models, and external representations of these models, are developed over several iterations and revisions within science practice, where the internal and the external interact and help change each other (Nersessian, 2010). The final internal models and related representations, which students are expected to learn in an integrated fashion, are often dense and opaque end-products, hiding the historical contexts and the problems through which they evolved. In chemistry, for instance, practitioners often use different models of matter, to study, understand, explain and/or synthesise substances through various chemical processes, often describing chemical phenomena (such as colour change) in terms of molecular level interactions. Chemical phenomena are understood at multiple levels, with the help of representations at each level, such as reaction mechanisms, molecular diagrams, graphs and equations. One critical aspect of learning chemistry is developing expertise over these MERs, often without an understanding of how and why these particular representations emerged, and why they are optimal. Similar expertise over MERs is required to learn and practice other disciplines, such as biology, physics, mathematics and engineering.

There is a vast literature that examines the learning of MERs, as well as the use of MERs in science practice, particularly integration and transformation of MERs – a skill termed representational competence (RC). These studies are widely dispersed, very often published in discipline-specific venues, and there is no well-articulated theoretical framework that helps integrate these disparate studies. A coherent theoretical framework to understand the problem of RC is critically needed, particularly given the emergence of new computational media that is helping develop novel external representations for learning and discovery (Chandrasekharan, 2016; Chandrasekharan & Nersessian, 2015), such as video games that embed computational models, and enactable representations of formal systems (Abrahamson & Sánchez-García, 2016; Kothiyal et al., 2014). These new media make possible new ways to integrate MERs, as well as integrate MERs and internal models, in turn facilitating new ways of thinking about phenomena and making discoveries (Chandrasekharan, 2016; Chandrasekharan & Nersessian, 2015). However, design guidelines for developing such media are yet to be developed, as there is no clear understanding of the nature of RC, the key skill the new designs seek to support.

In this paper, we review the existing literature on RC, and outline a theoretical framework that could help lead to design guidelines for the development of new media for science learning and discovery. Specifically, we argue for a distributed and embodied cognition account of RC, for three reasons:

- One, current models of cognition reject the classical information processing approach; mental processes are now understood as distributed and embodied. Models of RC are models of cognition, and thus need to incorporate this theoretical shift, particularly because MERs are external (thus distributed), and working with MERs require sensorimotor interaction (embodied interaction).
- Second, there is a parallel shift in the design of new computational media, where embodied controllers such as Leap Motion, Kinect, Real Sense and Virtual Reality are used to develop new learning

experiences (Abrahamson & Sánchez-García, 2016; Dickes, Sengupta, Farris, & Basu, 2016). This design approach requires understanding the role of embodiment in RC.
- Finally, the practice of science is now understood as distributed and embodied (Chandrasekharan, 2013, 2014; Chandrasekharan & Nersessian, 2015; Nersessian, 2010), and models of RC need to reflect this shift in our understanding of science practice.

A central component required to develop a distributed and embodied understanding of RC is a model of the way the cognitive system *interacts* with external representations, as opposed to the view that all external representations embed information, that this abstract information is isolated from the external structure and pulled inside by the cognitive system (somehow), and that cognition arises from the manipulation of this abstract information inside the head. Addressing this issue, recent work by Landy, Allen, and Zednik (2014) articulates a distinction between syntactic/semantic approaches and constitutive approaches towards symbolic reasoning. In the first approach, symbols in MERs are considered to be internalised by the cognitive system, and then processed fully inside, i.e. just using neural processes. In the constitutive account, the external symbols are part of cognition, and the external operations on them, as well as the sensorimotor processes involved in these operations, are part of the cognition process. This argument for the constitutive view gets further support from the fact that most scientific phenomena deal with entities not available to perception and action, and therefore the understanding of these entities is tightly intertwined with the external structures that stand in for these entities. The MERs thus play a twofold constitutive role in cognising these phenomena, as understanding these imperceptible entities would be impossible without them, and since MERs are external structures, operations done on them are a critical component of understanding the entities and processes they stand in for. Any account of RC thus requires taking a constitutive view of MERs, and this requires providing a model of the nature of this constitutive process, particularly the role played by external structures in changing cognition, to generate new ideas. Note that this constitutive view is inclusive, and does not deny the 'standing-in' (representational) role played by MERs. The representational role is in fact a central component of this constitutive approach.

The assumption that all cognitive processing is done just by neural processes, and/or is best done just using neural processes, is questioned by Kirsh (2010), who outlines seven ways in which the external aspect of external representations, and the sensorimotor interactions with external representations, contribute to cognition:

(1) They change the cost structure of the inferential landscape.
(2) They provide a structure that can serve as a shareable object of thought.
(3) They create persistent referents.
(4) They facilitate re-representation.
(5) They are often a more natural representation of structure than mental representations.
(6) They facilitate the computation of more explicit encoding of information.
(7) They enable the construction of arbitrarily complex structure; and they lower the cost of controlling thought – they help coordinate thought.

'Jointly, these functions allow people to think more powerfully with external representations than without. They allow us to think the previously unthinkable'. (Kirsh, 2010). This view suggests a more interactive way of understanding why and how MERs help advance scientific reasoning.

However, understanding RC, particularly to provide design principles for developing new media, requires moving beyond just the recognition of the cognitive power of external representations: it needs a model of how new kinds of imagination are made possible by the coupling of MERs with the imagination system (Chandrasekharan & Nersessian, 2015). This coupling is closely related to integration of MERs. Since MERs capture different aspects of a phenomenon, they need to be integrated by the learner to understand the nature of that phenomenon. Any account of how MERs are used in learning, thus, needs to account for this integration process, particularly the role played by interactions with MERs and the sensorimotor processes involved in this integration.

The above three aspects (constitution, power of external representations, integration of external representations with the imagination) are not addressed by current work in RC, except in some isolated specific cases. In this paper, we seek to address this gap in theory, by outlining a distributed and embodied cognition approach to RC, in two steps. First, we provide a comprehensive review of the existing work in RC, by examining the influential RC models (Section 2), and empirical studies (Section 3). A set of models and studies discussed under these two sections explicitly appeal to the classical information processing paradigm, while some other frameworks and studies implicitly assume classical information processing perspectives, but do not endorse this view explicitly. A third set of models and studies are neutral on the nature of MERs and RC. There is also a group of models and studies that subscribe to recent cognition theories such as distributed and/or embodied cognition. This categorisation based on subscription of theoretical models and empirical studies to larger models of cognition is captured in a chart. A discipline-based review is also presented for quick review, through boxes for each discipline, and a set of tables that brings together the diverse literature in a discipline-based categorisation. We then discuss the findings from this review (Section 4). In the second step, we outline a distributed and embodied account of RC (Section 5). This is a preliminary account, drawing on related theoretical and empirical work on the cognitive science of scientific discovery (Chandrasekharan, 2009, 2013, 2014; Chandrasekharan & Nersessian, 2015).

### 1.1. Review methodology

Three different modes were employed to collect articles for this review: (a) keyword search on the ERIC database, (b) keyword search on Google Scholar and (c) articles found relevant through cross-referencing. The following is a list of keywords used for methods (a) and (b):

> scientific representations, learning – multiple representations, RC, RC in biology, RC in science (then the word 'science' replaced with chemistry, biology, physics, mathematics, and engineering), multiple external representations in science. (the word 'science' then replaced with chemistry, biology, physics, mathematics, and engineering), multiple representations in science (the word 'science' then replaced with chemistry, biology, physics, mathematics, and engineering)

Articles found using these keywords were filtered based on their date of publication, relevance and major discipline. Only articles published after 1990 were read and analysed (with a few exceptional articles from before 1990 included due to their evident influence as well as frequent citations – e.g. Johnstone, 1982; Lesh et al., 1987, etc.). Further, only those studies/articles related to cognition research on multiple representations were included in the review. Articles exploring other dimensions, such as testing of a multi-representational user interface, use of multi-modal representations and social significance of multiple representations were not included. Also, investigations of a single representational system (for instance only drawing) were not included, unless the studies had wider implication/significance for the understanding of multiple representations.

### 1.2. Results

The review of the representational competence literature shows that there are very few studies that are based on the constitutive view, and most studies implicitly assume the syntactic/semantic approach and the classical information processing model. This is a vast literature, with many disparate studies, conducted in many different fields. The research on the use of MERs in science education is also very diverse, examining everything from children's 'scribbling' on paper, drawing of simple diagrams, making sense of the diagrams (reasoning), to complex modelling of scientific phenomena by practitioners and working memory models of representational transformations. The following four major findings emerged from our review:

### 1.2.1. Ambiguity in using the term 'representation'

Most of the literature uses the term 'representation' ambiguously to refer to either internal/mental representations or external ones, or both. Very few studies explicitly distinguish external representations from internal/mental representations. Given this ambiguity, the problem of how the external and internal representations interact is rarely discussed or examined, particularly in raising/lowering cognitive load (as the studies claim), or improving imagination, while thinking about scientific concepts.

### 1.2.2. MERs and RC differ across disciplines

The nature of MERs, particularly the roles they play in thinking, differs across science (chemistry, biology and physics), mathematics and engineering. Consequently, characterisation of RC differs across disciplines.

### 1.2.3. MERs present a general cognitive difficulty

Despite these differences, there is a consensus in the literature across disciplines that the difficulty in mastering MERs underlies many different learning difficulties. Also, studies on the nature of expertise show that, in a specific knowledge domain, understanding of representations, and the ability to generate and use MERs in an integrated fashion (for conceptualisation, discovery and communication) are indicative of expertise. These common features suggest that integration of MERs (in other words, RC) is a general cognitive difficulty. The general cognitive difficulty takes different forms based on discipline-specific MERs.

### 1.2.4. Focus on classical information processing models

The review shows that the existing accounts of this cognitive difficulty are mostly based on classical information processing accounts of cognition, emphasizing the notion of cognitive load, and a 'mental capacity' to handle the load. A central problem of such capacity models is that they shift the attention away from the nature of the internal and external representations, particularly the cognitive mechanisms involved in processing, interacting with and integrating MERs, and possible interventions based on such mechanism models. The focus on load and capacity also makes the process of RC development appear mysterious. Related classical information processing assumptions underlying studies include an explicit focus on working memory, centralised processing of information (indicated by terms such as translation, which suggest information from MERs is extracted in some form and then translated into another form), and hierarchical levels of processing.

Building on these findings, we outline an account of the cognitive mechanisms involved in MER integration, based on recent cognitive theories, particularly distributed and embodied cognition. This preliminary account makes explicit its departure from classical cognitivist assumptions, and is proposed as a general theoretical framework from which many interventions could follow, including new media, classroom interaction, inquiry, and manipulative-based teaching. We emphasize the distinction between internal and external representations, considering the two as dynamically coupled through constant interactions between the learner and external representations. Our focus is on how different external representations are integrated. But since this integration process is closely coupled with the formation of an internal model of the domain, our model also considers integration of MERs and internal models. We suggest that MERs are understood by learners through an 'incorporation' process, where they become part of, and thus extend, the cognitive system, as well as form and extend the internal model of the scientific domain. This incorporation process is considered to be driven by actions/manipulations performed on the external representations, as well as through exploring many states of the external representations. Further, actions on these MERs (overt as well as covert activation of the motor system) facilitate 'freezing' and 'unfreezing' the different states of MERs, and these operations play a central role in MER integration. We explore some of the implications of this mechanism-based account in developing interventions in science, engineering and mathematics, particularly interventions based on new-media.

## 2. Theoretical accounts of MERs and RC

A wide range of conceptual frameworks have been proposed to capture learning and cognition through MERs. We examine two kinds of such models: (1) models based on the relationship between the nature of a domain, MERs in that domain and cognition and (2) developmental models. Models under the former section are further categorised into three interrelated but different subsets: one set focuses on the nature of knowledge in a domain, particularly pertaining to the space–time scales/levels, a second category focuses on reasoning through MERs, and a third set concerns mechanisms of MER cognition. Developmental models, on the other hand, focus more on the process of learning using MERs through stages of development, and are either based on (1) or are independent in a broad theoretical sense.

### *2.1. Models of MERs and cognition*

#### *2.1.1. Relationship between the nature of domain and MERs in that domain*

Different scientific domains (biology, chemistry, physics, etc.) differ from each other in certain fundamental aspects, such as the nature and scale of problems, investigation methods and data. These differences reflect in the nature of MERs used across these disciplines, and models of RC.

One of the first models of the relationships between the nature of a scientific domain, the MERs that constitute it, and a learner's interaction with those MERs, was proposed by Johnstone (1982). The model examines the visual-perceptual nature of representations used in science, particularly in chemistry. Chemistry MERs include the periodic table, chemical equations, graphs, molecular formulas, diagrams of experimental set-ups, diagrams depicting molecules, etc. Each of these conveys different information on chemical entities and phenomena. Johnstone's model, known as the model of 'three thinking levels', describes the way the discipline of chemistry is conceptually organised around these MERs. Box 1 provides a quick summary of the discussion on the nature of chemistry MERs, as well as a review of MERs and RC in chemistry education literature.

According to the Johnstone, knowledge in chemistry can be viewed at the following three levels (Figure 1):

(a) Descriptive/functional/macro level, which deals with handling of materials, descriptions of phenomena and their properties, such as colour, flammability and density.
(b) Representational/symbolic level, which deals with representations of chemical substances and phenomena using symbols, formulas, equations and conventions.
(c) Molecular and explanatory/micro/sub-micro level, which captures the structure of chemical substances and phenomena, mechanisms of reactions, and the molecular/atomic interactions and changes that underlie chemical phenomena.

The model considers MERs in chemistry as distributed across the three levels of thinking, and learning as well as doing chemistry requires, in this view, simultaneously processing the information gathered from MERs at all the three levels (this is characteristic of expertise).

Supplementing this model of 'three thinking levels' with Baddeley's model of working memory, Johnstone (1991, 2000) attributes students' difficulties in learning chemistry to the way this schema (the conceptual organisation of chemistry) interacts with the limited capacity of the human working memory. According to Johnstone, the three-level schema puts significant load on a student's working memory as she attempts to understand a chemical reaction in terms of its equation and/or a graph (symbolic level) as well as the molecular mechanism (molecular level) of the reaction. As a result of the load, and the limited working memory capacity, students often ignore important features of the phenomenon, concentrating only on parts of it.

Several other models in chemistry education research attempt to conceptually organise chemical knowledge. Jensen (1998), for instance, replaces 'macro' level with 'molar' (referring to the perceivable stoichiometric ratios of chemical substances handled and used in carrying out reactions), retains the molecular level, and defines a third level called the electrical level, at which chemical phenomena

**Box 1**

**Nature of chemistry:**

Chemistry concerns the study of entities and phenomena which often cannot be observed directly at the level at which they occur (e. g. dissociation, neutralization, equilibrium dynamics, etc.). Learning and doing chemistry involves frequent use of different kinds of indirect representations, such as chemical equations, graphs, molecular formulas, diagrams of experimental setups, diagrams depicting molecules, etc., which help connect us indirectly with the actual chemical phenomena. Our thinking/understanding about chemical phenomena is essentially guided by these representations, and the nature of these representations influences teaching/learning/doing chemistry.

**Central external representations identified in previous research:**

Periodic table, chemical equations, concentration/energy graphs, molecular diagrams, static and dynamic visualizations, graphical projection for organic molecules (e.g. Fischer's projection formulas), informal and formal gestures for representing spatial configurations and conformations of molecules and stereo-isomers, animations and simulations.

**Representational competence (key definitions identified in the previous research):**

Integrating MERs to understand chemical phenomena, inter-relating and transforming between MERs in a dynamic fashion, generation of appropriate MERs.

**Existing experimental approaches to study representational competence:**

Analyzing students' problem solving process, microgenetic studies, ethnographic observations of professional chemists, expert-novice comparison, prior knowledge & representational competence correlation, computer interface testing, eye-tracking

**Major results from previous research studies:**

(a) Various learning difficulties in chemistry can be attributed to difficulties in understanding, handling and using MERs, (b) prior knowledge plays a key role in developing representational competence, but it does not guarantee it, and (c) while inter-relating MERs, novices rely upon surface features, whereas experts use chemical principles.

**Main theoretical frameworks proposed in the previous research:**

Dual coding model, Johnstone's model of three thinking levels (macro level, sub-micro level and symbolic level) based on the classical information processing model of cognition, particularly Baddeley's working memory model. Some studies combine Johnstone's model with distributed and situated cognition approaches.

**Main existing intervention approaches:**

(a) Development of visualization software that seek to lower working memory load, by simultaneously presenting on screen chemical equations, concentration and/or energy graphs, molecular-level animations and laboratory experiment video (e.g. SMV Chem, visChem, 4M:Chem, EduChem HS, etc.), (b) use of physical models, (c) laboratory-centered pedagogy, (d) MER centred problem-based curricula, and (e) integration of MERs through manipulable interfaces that display the interrelation between different representations.

**Box 1.** Quick review of research on MERs and representational competence in chemistry.

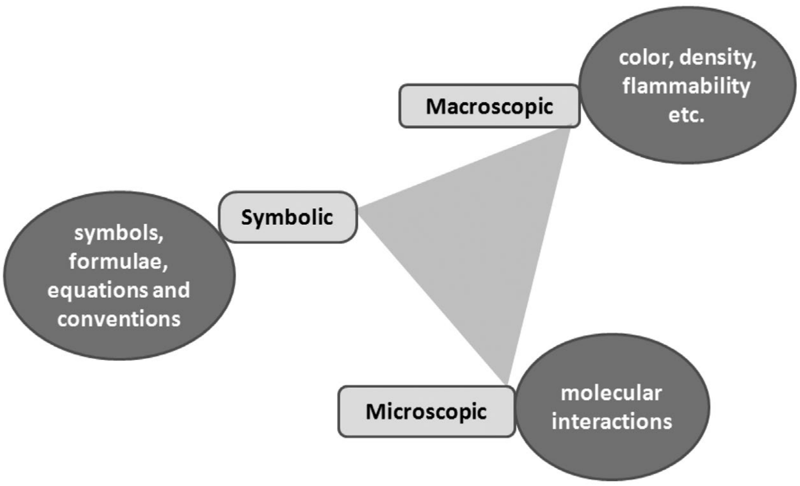


**Figure 1.** Johnstone's model of three thinking levels.

**Box 2**

**Nature of biology:**

Knowledge in biology is distributed across several levels of organizations that are used to characterize biological systems. For instance, the structure and functions of an organism as a complex system are dependent on the structure and functions of the numerous cells it is made of. The cell itself is a tiny semi-autonomous, self maintaining system composed of biomolecules and their assemblies, where various biochemical pathways such as translation, glycolysis, and Calvin's cycle are understood in terms of their respective locations in specific parts of the cell organelles (ribosomes, mitochondria, stroma of chloroplasts). MERs used in biology parallel this multi-level structure. Representational competence problem is more complex in this domain than in chemistry. Research on representational competence in biology is relatively recent in comparison to chemistry, physics, and mathematics.

**Central external representations identified in previous research:**

Biochemical pathways, structural (configuration & conformation) representations of biomolecules, DNA/polypeptide helices (right/left handed), replication, transcription and translation mechanisms, planar depictions of animal body parts (ventral, dorsal), anatomical planes/sections (lateral, cross, sagittal, temporal, transverse, etc.), developmental and evolution time-scales, phylogenetic trees, punnett squares, Hardy-Weinburg equation, computational models of predator-prey dynamics and similar complex systems. These representations cut across multiple organizational levels, from molecules to cells, tissues, organisms, communities and ecosystems.

**Representational competence (key definitions identified in the previous research):**

Developing an integrated understanding of MERs that are distributed across different levels of organization.

**Existing experimental approaches to study representational competence:**

(a) Student misconceptions in relation to the multiple levels of organization, (b) students strategies of interrelating MERs distributed across the different levels of organization, (c) experts-novice differences, (d) eye-tracking, (e) relation between prior knowledge and MER transformation/interlinking.

**Major results from previous research studies:**

(a) Generating MERs and relating them to previously encountered concepts helps in problem solving, (b) previous knowledge is strongly related with frequency of making transitions between MERs, (c) the static nature of MERs, which embed dynamic phenomena, could critically limit interrelating MERs.

**Main theoretical frameworks proposed in the previous research:**

CRM model, Kaptejin's model of three levels, and the cube model supplemented by Baddeley's working memory model.

**Main existing intervention approaches:**

Visualization software, mainly seeking to connect different levels of organization, guided by the information processing model of cognition. There also exist manipulable Netlogo models of complex, self organizing biological systems. Pedagogical interventions include problem-based learning (especially in genetics, biochemistry and medicine) and inquiry-based learning.

**Box 2.** Quick review of research on MERs and representational competence in biology.

are explained using subatomic particles (such as electrons) and their dynamics. Ben-Zvi, Eylon, and Silberstein (1988) propose that single-particle modelling is sufficient to describe chemical properties of substances (but not physical properties). They then suggest that the sub-micro-level be split into single-particle and multi-particle sub-micro-levels of understanding chemical processes. A distinction between symbols used to denote chemical substances, and the numbers in stoichiometry, kinetics and mechanisms has also been proposed (Garforth, Johnstone, & Lazonby, 1976; Nakhleh & Krajcik, 1994; Savoy, 1988).

In biology, Kapteijn (1990) proposed a framework of biology MERs in relation to (a) the levels of biological organisation, as well as (b) observability of the MERs, i.e. one's ability to see entities and phenomena. Keptejin's model, similar to the Johnstone's model, has three distinct levels of MERs, viz. macro (organismic), micro (cellular) and molecular (biochemical). According to this model, the ability to visualise entities and phenomena at all the three levels limits students' understanding of biological phenomena. Box 2 provides a quick access to the nature of biology MERs, as well as a review of MERs and RC in the biology education literature.

Tsui and Treagust (2013) recently proposed a more comprehensive framework of the conceptual organisation of biology, termed the cube model, which proposes a three-dimensional knowledge structure. In this model, knowledge in biology is spread across three different, but interdependent, dimensions and learning in biology is marked by one's progress along these dimensions (Figure 2):

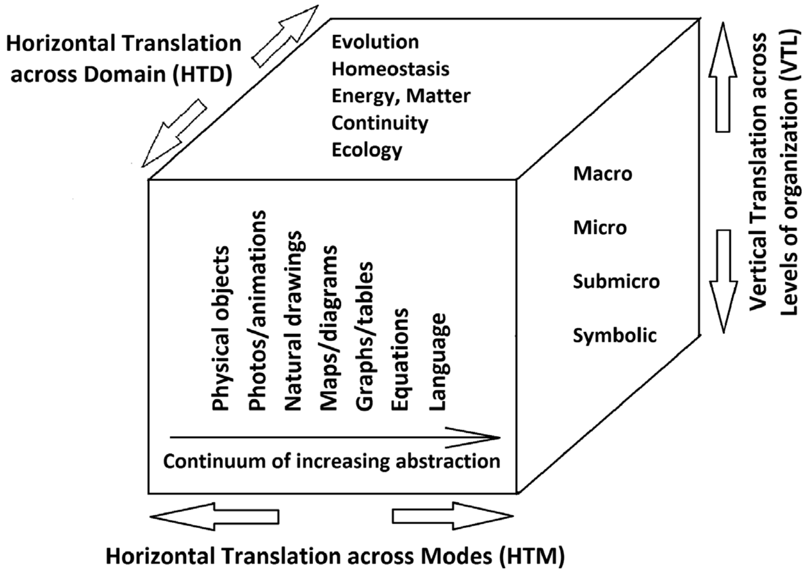


**Figure 2.** The cube model.

(a) HTM: Horizontal Translation across Modes of representations, 'along a continuum of representations with increasing abstraction from real-life objects and actions to human language'.
(b) VTL: Vertical Translation across Levels of representations 'from the symbolic level (explanatory mechanisms), the sub-micro-level (molecules), the micro-level (organelles and cells), and the macro-level (tissues, organs, systems, organisms, populations, and so on)'.
(c) HTD: Horizontal Translation across the Domain knowledge of biology, i.e. across evolution – homeostasis – energy – matter and organisation – reproduction and genetics, etc.

Items from VTL can have one-to-many relationships with items from HTM, but not necessarily the other way around. For example, we often associate the term 'macroscopic' (VTL) with 'observable' (corresponding to worldly objects/actions, maybe even photographs/animations, essentially items along HTM). Also, graphs, tables and equations can all be counted under symbolic level or representations. However, equations would be strictly symbolic, and cannot be under the microscopic level. The VTLs are categories of representations similar to the levels of thinking, whereas HTMs are various modes through which information across those categories is obtained, presented and communicated. Although it is simple, comprehensive and unified, the cube model has limitations in capturing phenomena occurring over large temporal scales, such as evolution (Tsui & Treagust, 2013).

In mathematics, one of the most discussed and widely used conceptual frameworks is the Lesh Translation Model (Figure 3), which is a network model developed to investigate student-generated representations and (information) translations between multiple representations. The model proposes that knowledge in mathematics is structured across five different, but interrelated and interconnected modes of representations, viz. (1) concrete/manipulable objects/situations (e.g. physical manipulatives such as tangram), (2) pictorial representations such as 2D/3D diagrams, (3) real-life contexts (e.g. acts of addition, sharing, etc.), (4) language (e.g. usage of mathematical terms such as 'addition' and 'subtraction') and (5) written symbols (symbols denoting mathematical operations). Mathematical understanding is reflected in the ability to represent mathematical ideas in multiple ways across these five representational modes, and also in making connections and translations among them (Lesh et al., 1987). From a pedagogical perspective, the term 'translation' emphasises interrelating information extracted from one representation with information from another. An expert would be fluent in translating between these proposed representational modes. The model has driven the conceptualisation

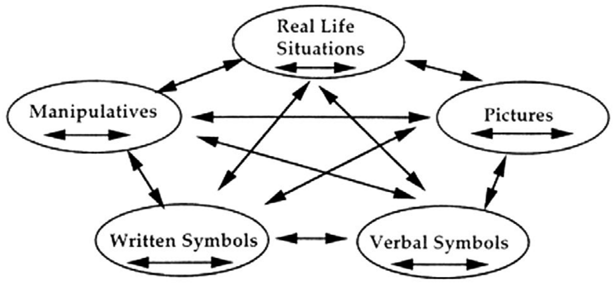


**Figure 3.** Lesh translation model.

**Box 3**

**Nature of physics**:

Physics aims to understand universal phenomena in terms of matter and its motion. It involves explaining the nature of physical worlds as small as atoms and as large as the universe itself, using abstract concepts such as force and energy, and time and space. Most frequently used MERs by physicists include: motion/kinematics graphs, mathematical equations, equations of laws (Ohm's law, Newton's laws, etc.), ray diagrams, force-body diagrams, circuit diagrams, Feynman diagrams, astronomy models, gestures (right/left hand thumb/grip rule, gestures in kinematics and astronomy), computational models, etc.

**Central external representations identified in previous research:**

Motion/kinematics graphs, mathematical equations, equations of laws (Ohm's law, Newton's laws, etc.), ray diagrams, force-body diagrams, circuit diagrams, Feynman diagrams, astronomy models, gestures (right/left hand thumb/grip rule, gestures in kinematics and astronomy), computational models.

**Representational competence (key definitions identified in the previous research):**

While integration of MERs is necessary for physics learning, representational competence in physics is typically characterized by the ability to boil down all representations to abstract mathematical equations and working with them. Understanding and working at the level of abstract equations implies that one has an integrated understanding of MERs in physics.

**Existing experimental approaches to study representational competence:**

Expert-novice comparisons of problem representations/comprehension, representational transformation between mathematical and real-world physical phenomena/entities, ability to generate and manage MERs while problem solving; meta-representational competence.

**Major results from previous research studies:**

Experts are good at situating problems in relation to physics principles/laws, whereas novices rely on literal meanings of the problem statements. Experts are better able to generate MERs and fluently (yet less frequently) shift between them while problem solving.

**Main theoretical frameworks proposed in the previous research:**

Meta-representational competence, native competence, expert-novice comparison model

**Main existing intervention approaches:**

Computer simulations (PhET, Netlogo), problem-context-based curricula, computer visualization, and virtual laboratory.

**Box 3.** Quick review of research on MERs and representational competence in physics.

and development of a set of specific activities (called model eliciting activities) in mathematics and engineering pedagogy (Moore, Miller, Lesh, Stohlmann, & Kim, 2013).

Due to the intertwined nature of physics (Box 3), mathematics (Box 4) and engineering (Box 5), the Lesh translation model is equally applicable to MERs in physics and engineering. Manipulable models and prototypes of physical and engineering objects, free body diagrams, acts of navigation and motion, use of terms such as 'speed' and 'distance' in language, and symbols denoting physical properties of objects and phenomena such as 'force' and 'energy', are some examples of physics and engineering MERs belonging to the five modes of representations, respectively.

As opposed to Lesh et al.'s network model, Roth and Tobin (1997) suggest a linear cascade model to explain the relationship between physics learning and practice, and the nature of physics MERs. This model emerged from an investigation to understand how teachers (university professors) use and translate between MERs while teaching in a physics class, and how this relates to students' difficulties

**Box 4**

**Nature of mathematics:**

The term 'multiple representations' has slightly different (and perhaps additional) connotations in mathematics than 'multiple representations' in the sciences. Multiple representations in mathematics do not necessarily mean representations belonging to multiple media/modes. For example, '2 + 6' and '8' are both in symbolic form, but are different kinds of representations for the same concept. This makes representational competence in mathematics more complicated to define and characterize. Two categories of MERs are suggested in the literature (Lesh, Post & Behr, 1987); opaque and transparent. Consider the representations 'eight' and 'a picture of four black ovals', these convey the exact amount of information about the entity they represent (here a number). However, consider the representation '2x4' which leads to the number/quantity '8', but in addition conveys that the '8' is a multiple of 2 (or 4). This information is not conveyed by the earlier two representations. In this case, the earlier two representations are transparent, whereas the latter is an example of opaque representation. Students often ignore the transparent or opaque nature of representations when they are asked questions based on those representations. Such representations are not meaningful to students until derived into other representations. The use of MERs in mathematics starts very early, right from reciting of numbers and counting (connecting real world objects and numbers) to learning mathematical operations such as addition, subtraction and their real world meanings (through terms like 'lending' money or 'borrowing' it), and gaining expertise in the use of symbols and operators. MERs help mediate different facets/meanings of a mathematical concept, such as the concept of a number. An integrated understanding of these facets, mediated by MERs, is central to learning and doing mathematics.

**Central external representations identified in previous research:**

Numbers, the real-number line, base-ten numerals, algebraic notations/equations, trigonometric models, Cartesian coordinate system, graphs, matrices, sets, geometries, metric spaces, programs, algorithms, logic, embodied and standardized measurement scales and tools (such as the ruler), real-world activities (borrowing, lending, interest, selling areas of land, etc.)

**Representational competence (key definitions identified in the previous research):**

The ability to shift between different MERs, particularly spatial and numerical representations, and transform between them.

**Existing experimental approaches to study representational competence:**

(a) Abilities to inter-relate abstract MERs and concrete situations represented as diagrams, (b) difficulties in relating number/quantity and space in measurement tasks, and (c) reasoning during problem solving based on generation and interpretation of MERs, (d) ethnography.

**Major results from previous research studies:**

Students lack the ability to develop/find equations relating to concrete situations. They also have difficulty integrating space and numerical value. However, students are good at inventing and using criteria to generate, use and interpret MERs to solve concrete problem situations.

**Main theoretical frameworks proposed in the previous research:**

Level of abstraction in MERs, distinction between abstract and concrete sets. Expert-novice comparisons, Lesh translation model and Duval's stage/level models of mathematical competence.

**Main existing intervention approaches:**

Model eliciting activities and problem-based approaches. Forcing of frequent transformations between abstract, realistic-concrete and language based MERs, computer simulations (GeoGebra, Netlogo), virtual/physical manipulatives (killmath)

**Box 4.** Quick review of research on MERs and representational competence in mathematics.

in understanding the topic ('motion of a rolling ball on an inclined plane'). The authors propose a continuum of abstract and concrete representations, generalised from findings from the nature of MERs used in the classroom, to explain how the types of MERs in science, mathematics and engineering relate to student difficulties in interrelating information embedded in them. The continuum has more concrete representations (such as photographs and pictures of real-world objects/phenomena) on one end, more abstract representations (such as equations representing relationships between those worldly objects and phenomena) on the other end, and other representations placed in between, based on their abstract/concreteness. All these representations are separated by ontological gaps, and the distance between any two representations on the continuum is proportional to the ontological gap between them, which is in turn proportional to the difficulty to translate between them. In this view, students have conceptual difficulties because they lack an understanding of the translation process across items on the cascade.

Johri, Roth, and Olds (2013) refine this cascade model (Figure 4), in the context of engineering design, focusing on the relationship between the world and language (and/or thought), where design moves through a series of representational transformations, which bring the world and the word closer. The

**Box 5**

**Nature of engineering:**

Engineering involves designing, building, and maintaining different kinds of materials, systems, and processes, as well as a diverse set of structures such as machines, devices and buildings. Engineering activities heavily rely on the iterative generation and use of multiple representations (e.g. materials, inscriptions, sketches and drawings, functions, equations and graphs, prototypes), mathematical and physical models, and simulations. Inscriptions and MERs are crucial particularly to material activities. These MERs mediate designing/creating constrained real-world things, by making use of abstract ideas and operations (design mode), as well as connecting abstract operations with the behavior of real-world entities (testing mode), in a streamlined fashion. The ability to use MERs to achieve engineering ends is referred to as 'representational fluency' (equivalent to representational competence). Given its interdisciplinary nature, MERs in engineering are vast and diverse, and therefore the integration problem in engineering is more complex, compared to science.

Representational competence studies in engineering education are limited and relatively recent, and are mostly ethnographic investigations coupled with approaches from cognitive science. The next section discusses at some recent empirical work. The later sections discuss theoretical frameworks to characterize representational competence and student difficulties with MERs in engineering.

**Central external representations identified in previous research:**

User requirements, sketches, drawings, functions, equations, materials, designs, models, prototypes, inscriptions, computer simulations, algorithms, programs.

**Representational competence (key definitions identified in the previous research):**

Integration of various abstract, concrete and spatio-temporally distributed MERs, often in the context of real-world problem-solving.

**Existing experimental approaches to study representational competence:**

(a) Socio-cognitive aspects of engineering, design and technology, (b) ethnographic descriptions of real-world engineering problem solving, (c) how science, mathematics and engineering disciplines could be integrated by exploiting distributed MERs and concepts across these disciplines.

**Major results from previous research studies:**

Integration of MERs is central to engineering learning and practice; MERs help offload information, and also helps add details to the design. Concepts are distributed across MERs, and so is engineering practice.

**Main theoretical frameworks proposed in the previous research:**

(a) Representational-chain model, (b) Lesh translation model, (c) Distributed and situated cognition

**Main existing intervention approaches:**

Model-eliciting activities (a problem-based pedagogy), computer mediated MER integration, visualizations and simulations, STEM integration activities.

**Box 5.** Quick review of research on MERs and representational competence in engineering.

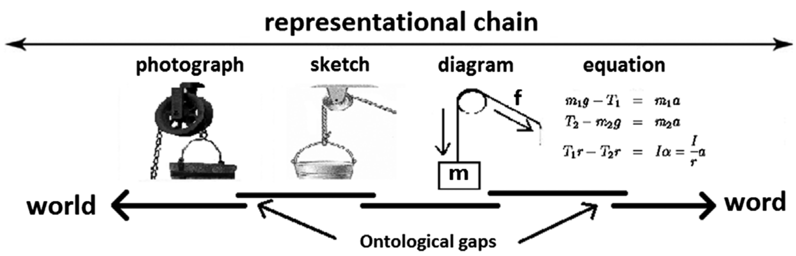


**Figure 4.** The representational chain model.

model situates words and abstract symbols on one extreme, while world (as experienced by a cognitive agent) on the other extreme of a continuum of representations. Starting with the notion of ontological gaps (Roth & Tobin, 1997) between the worldly phenomena and their representations, Johri et al. (2013) argue that the representational translations are crucial in bringing the world and the word closer. In the natural sciences, this movement of the cognitive agent through a continuum of representations happens from the world to the word; whereas in engineering practice, it is the other way round (Johri et al., 2013; McCracken & Newstetter, 2001). In the context of education, Johri et al. state that difficulty in transformations between the kinds of representations results in difficulties in learning.

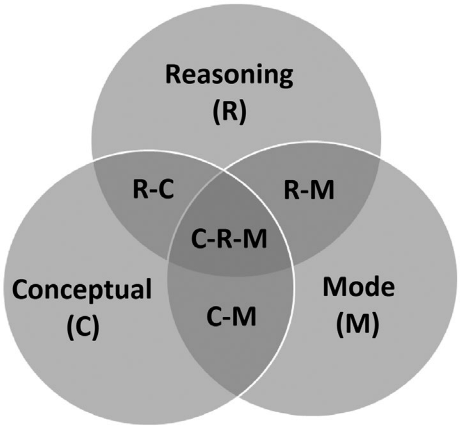


**Figure 5.** CRM model.

### *2.1.2. Reasoning and MERs*

The frameworks discussed under this section model students' interpretation of MERs and their reasoning in relation to scientific concepts.

Different external representations present different aspects of the world, and thus, serve different functions in cognition, communication and other activities. Ainsworth (1999, 2008) presents three different functions of multiple representations – (a) they are complementary to each other (as different representations provide different perspectives about the same phenomenon and/or entity), (b) one external representation may constrain the process of interpreting another (as a result of familiarity with it) and (c) multiple representations together help conceptual understanding (through representational integration). This proposal can be interpreted as suggesting possible ways in which a learner may use multiple representations to understand scientific concepts and reason about them. The model is employed by education researchers across many scientific disciplines (e.g. Won, Yoon, & Treagust, 2014), for designing interventions as well as understanding the cognitive underpinnings of processing and integrating multiple representations.

Students' ability to interpret MERs in biochemistry is the focus of the model of Schönborn and Anderson (2009). It has three main intertwined components: concepts, reasoning and modes of representations (Figure 5). According to the authors, this description of different abilities provides a framework for classifying expert ways of reasoning (i.e. characterisation of RC) and analysing students' reasoning difficulties. For instance, experts are good at integrating any components the model describes, because they have the necessary conceptual knowledge, reasoning abilities and understanding of MERs to convey their conceptual knowledge or reason about phenomena and/or entities. Learning can be understood as development of connections between these components of the model, and a learner may exhibit the use or combination of any two or all the three components of the model (reasoning based on concepts, representation-mediated reasoning, relationship between representations and concepts embedded in them). The authors emphasise reasoning using MERs (modes of representations) in learning, given the central role of representations in science cognition. The model suggests various abilities that characterise competence in science. However, from an assessment perspective, it generates a large number of possible abilities, to assess each of which is difficult and time-consuming.

An alternative view is presented by Pape and Tchoshanov (2001), who explicate the distinction between internal and external representations, and recommend that thinking and reasoning through representations, in the context of mathematics, is a result of the interaction of (a) internalisation of external representations and (b) externalisation of internal/mental images. In learning, the mental images of primary mathematical concepts (such as addition, say, using base-10 blocks) are gradually associated with external representations for these concepts (such as '+'). Also, a key aspect of RC in mathematics is the ability to associate abstract mathematical content with physical representations

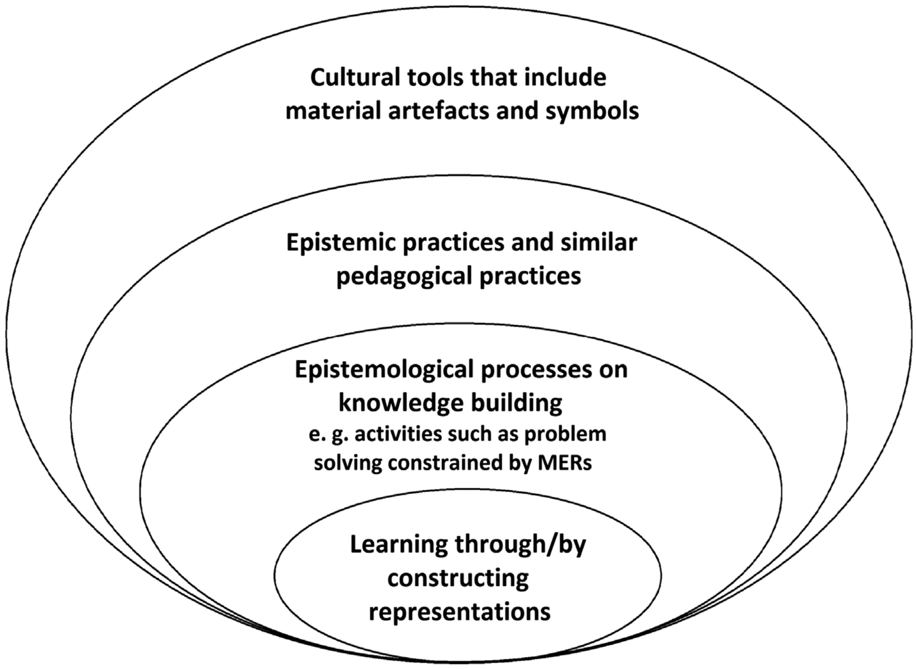


**Figure 6.** RCA model.

and vice versa. However, evidences of trade-offs during learning, between grounded (non-abstract, real-world representations, such as a word problem) and abstract mathematical representations (such as algebraic expressions drawn from word problems), have been reported in the literature (Koedinger, Alibali, & Nathan, 2008). The trade-off exists because there are cognitive costs to using the two types of representations (grounded and abstract). For example, as mathematical content to be learned gets more complicated, thinking in abstract representations becomes necessary, even though this is difficult.

An important contribution of this research stream is the discussion of the relationship between external and/or grounded and internal and/or abstract representations, which is overlooked by previously discussed models, and education research in general, despite being critical in understanding representational transformations, translations, coordination and reasoning (processes most of the models examine, but not from the perspective of the external–internal interaction).

The Representational Construction Affordances (RCA) model (Prain & Tytler, 2012), implicitly assumes this internal–external representation distinction, and focuses on the relationship between the act of generating representations and artefacts of different kinds in scientific reasoning and conceptual understanding. RCA model (Figure 6) – a Venn diagram of layered ovals of different sizes, with smaller oval(s) nested into larger oval(s) – concerns the relationships between broad and specific meaning-making practices in science around representational construction. The largest oval/layer signifies all the general material (instruments and artefacts) and symbolic tools (language, mathematics, gestures) offered by a culture. These general tools embed relatively specific representations (second oval) concerning epistemic and pedagogical practices around different knowledge systems (thought to be built on top of the general tools). Nested within the first two ovals (representational levels) are even more specific representational tools and practices concerning practice and pedagogy of science. The model is a pan-domain model, and presents how representations 'productively constrain meaning-making practices in science and in science education, taking into account the interplay of diverse cultural and cognitive resources students use to achieve this meaning-making'. Representational fluency or flexibility can be understood as the ability to fluidly move between the general and specific representational systems as required to facilitate meaning-making. The authors stress on the meaning-making point, and argue that a large fraction of the reasoning processes around MERs is informal in nature, i.e. not based on formal logic or other language-based systems (see also Tytler & Prain, 2010). Tytler, Prain, Hubber, and Haslam (2013) support this argument further by presenting case studies of students challenged to construct representations in order to solve problems on structure-function relationships in biology. They show that during the problem-solving process, students use visual and other non-formal modes of reasoning, along with linguistic forms of reasoning. Such informal modes of reasoning may be at the heart of MER integration, and thus, as Tytler et al. indicate, may have significant teaching–learning implications.

In the distributed cognition view (e.g. Kirsh, 2010), MERs are integral to the cognitive processes of an agent, and there is a continuous, dynamic interaction between the agent's internal and external representations. The distributed cognition approach revolves around two core principles; first that 'people establish and coordinate different types of structure in their environment' and 'people offload their cognitive effort to the environment whenever practical' (Aurigemma, Chandrasekharan, Nersessian, & Newstetter, 2013). Aurigemma et al. (2013) extend these two core principles to propose a model of the engineering design process, where the transformation process (between and among multiple representations) rely not only on the dynamic interactions between the internal and external representations, but also on the representation building and other actions that the agent engages in. Building of external representations, in this view, not only offloads cognitive effort, but adds detail and constraints to the mental model and the reasoning of the agent, which would otherwise (as advocated by the classical information processing theories), run only in the head, and lack these details.

### *2.1.3. Mechanism of MER cognition*

The models discussed in the above two sections focused more on the classification of external representations, and little on the mechanism of how the different kinds of MERs interact with a learner's mind. The frameworks reviewed below focus on the latter.

Wu, Krajcik, and Soloway (2001) propose a model of RC, examining the possible cognitive connections a learner could make between different available information sources, particularly external representations in chemistry. Informed by the general dual coding theory in cognition by Paivio (1991; elaborations and other versions by Mayer, 2005; Schnotz, 2002; Sweller & Chandler, 1991), the model implicitly assumes internal/mental representations and the external representations as distinct entities, and suggests that a cognitive system can be roughly represented into a $2 \times 2$ matrix (Figure 7), made up of four different subsystems: a conceptual system which is represented either (a) externally or (b) internally; and similarly, a visual system represented either (c) externally or (d) internally. The authors empirically verify that three specific kinds of cognitive connections are possible for a learner between her conceptual system and representations. The external and internal conceptual systems are connected (connection 1), as are the external and internal visual systems (connection 2). Moreover, the active learner also makes a connection between the internal conceptual and internal visual systems (connection 3). For instance, when a learner encounters an external conceptual stimulus, she actively interprets it (internally represents, connection (1)). Similarly, the external visual stimulus is also interpreted (internally represented, connection (2)). Often critical is the third connection, the connection between the internally represented conceptual and visual systems (Wu et al., 2001). Difficulties or errors in any of the three connections lead to difficulties in teaching–learning chemistry. This model is best understood as a model of the interaction between MERs and cognition than a model of levels.

Along similar lines, Schnotz (2002) describes a linear process of how MERs relate to cognition. According to his model, a learner initially perceives external representations (graphics or text) and creates a surface feature-based visual representation in the mind. This surface-feature-based mental representation of an external representation or model is then mapped on to common features from other mental representations of external representations/models, which consolidates into a mental model of the subject matter (Schnotz, 2002). Such a mental model is more abstract than the surface-feature-based visual representations and is incomplete, erroneous or absent in novices, as their internal representation remains at the visual level due to lack of prior knowledge.

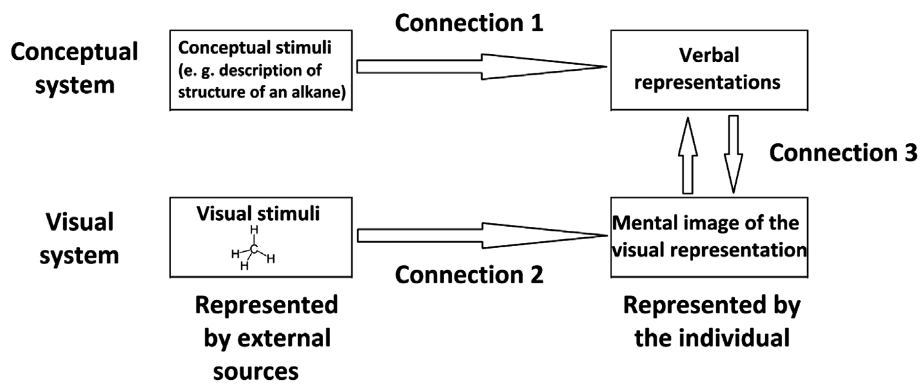


**Figure 7.** Wu et al.'s (2001) mechanism model of learning through MERs.

Based on an empirical study, Briggs and Bodner (2005) propose a model of problem solvers' ability to visualise molecules in a mental rotation task performed on an organic molecule. The results are interpreted in the form of a framework, which suggests that different components of a mental model are at work while handling multiple representations in organic chemistry. Four of the mental model components are: static representations viz. referents (physical objects), relations (spatial relation between referents; Gilbert, 2005), rules/syntax (order of referents guided by conceptual knowledge) and results (outcome/product of visualisation). Another component is dynamic, and is rather an operation (e.g. visualisation, rotation) performed on the static representations (Briggs & Bodner, 2005). Expert-novice differences can be explained on the basis of differences in static components, rules/syntax (conceptual understanding) and working memory. Unlike previous models, this model assumes the internal representations to be dynamic. However, the relationship between static and dynamic components is not clear. It is also not clear how the model would accommodate referents, relations and results that are dynamic in nature. The notion of conceptual knowledge is not clearly captured in the model, and the nature of conceptual knowledge could itself be dynamic than static.

### 2.2. Developmental models of RC

The focus of frameworks presented in this section is the process of RC development. These models may be informed by one or multiple theoretical assumptions discussed in the previous sections, and hence can be complementary to those models.

Dreyfus (1991) provides a linear stage model based on the number and complexity of representations simultaneously used by a learner. The model proposes that MERs mediate the process of learning, which passes through four sequential stages: (1) using single representations, (2) using more than one representation in parallel, (3) making links between the representations used in parallel; and (4) integrating representations as well as flexibly moving between them. The author argues that the processes of representation and abstraction are complementary processes moving in opposite directions. In other words, the act of representation is parallel to externalisation, while abstraction connotes internalisation.

A more sophisticated account of learning with multiple representations in mathematics is provided by Duval (2006). This model maintains that coordination between at least two representational forms, termed as registers, is necessary for comprehension of mathematical concepts. There are four such representational forms/registers: natural language, figures and diagrams, notation systems (symbols) and graphs. Learning with multiple representations involves students gaining more control over these registers. A learner initially stays within one register (e.g. carrying out calculations in only one notation system), then moves to conversions, where she changes the register (e.g. using notations/symbols like '+' to represent 'addition', a mathematical relationship originally described in language/words) and then finally achieves coordination among multiple registers.

Goldin and Kaput (1996) also provide a three-stage process of development of RC in mathematics. The stages are: (a) inventive-semiotic stage, where a learner is introduced to new characters of a representational system (for instance numbers/counting) that symbolise aspects of familiar systems such as a real-life situation; (b) the use of this system as a template to learn a more sophisticated system of rules for the new symbol-configurations (for instance, the concept of a number) and variations; and, (c) the new system, once learned and practised with, becomes independent and detached from the earlier system of representations (for instance, doing arithmetic/algebraic exercises). The third stage indicates abstraction, and is particularly critical in characterising RC in mathematics since mathematicians often operate in the world of abstract entities.

An influential model of the different abilities of experts and learners working with MERs is given by Kozma and co-workers, who coined the term 'RC', to describe

> a set of skills and practices that allow a person to reflectively use a variety of representations or visualisations, singly and together, to think about, communicate, and act on chemical phenomena in terms of underlying, aperceptual physical entities and processes. (Kozma, 2003; Kozma, Chin, Russell, & Marx, 2000; Kozma & Russell, 1997, 2005; Madden, Jones, & Rahm,2011)

The authors characterise RC in terms of following specific skills in the context of chemistry: (a) using representations to describe chemical phenomena; (b) generating and/or selecting and explaining appropriate representations for a specific purpose; (c) identifying and analysing different features of representations; (d) comparing and contrasting different representations and their information content; (e) making connections across different representations, mapping features of one type of representation onto those of another, and explaining the relationships between them; (f) understanding that the representations correspond to phenomena but are distinct from them; and (g) using representations in social discourse to support claims, draw inferences, and make predictions (Kozma & Russell, 2005).

Several researchers have argued that students' RC is often underestimated, despite reports suggesting difficulties in generation, selection, coordination and general handling of MERs among students (e.g. Izsák, 2011; Kieran, 1981; Leinhardt, Zaslavsky, & Stein, 1990). Students exhibit better competence than previously thought (diSessa, 2004; diSessa, Hammer, Sherin, & Kolpakowski, 1991; diSessa & Sherin, 2000). Preliminary research investigating the nature of untutored native competence among students in terms of content knowledge (whether inarticulate intuitions or articulable/potential principles), sources of such knowledge and the possibilities of refining this knowledge indicate that students' capabilities with representations were often underestimated by prior studies (diSessa & Sherin, 2000). Students are capable of having deep and rich, although intuitive, ideas about dealing with and making sense of external representations in their own ways. This competence is referred to as 'native competence, or meta-RC' by diSessa and Sherin (2000), and constitutes the following abilities: (a) invent or design new representations, (b) critique and compare MERs, for their appropriateness and adequacy, (c) understand various functions of representations in context, and how representations serve such functions in that context, (d) explaining representations and (e) learning new representations quickly with minimal instructions (diSessa, 2004). The notion of meta-RC is different from RC in the following way: it is concerned with whatever students know about the act of representation and its products (meta-representation); it does not focus on representations used for instruction in a domain, or the standard school modes of reproduction and interpretation (diSessa & Sherin, 2000).

### *2.3. Summary*

In this section, we reviewed important theoretical frameworks for RC, categorised under two major themes: models of RC concerning MERs and their cognition, and the developmental models of RC. Within the former category of models, we saw three different sets of models: one captures the relationship between nature of a domain, MERs in that domain, and cognition (model of three thinking levels and its versions, cube model, Lesh translation model, model of ontological gaps and representational chain model); the second captures how students reason about MERs (Ainsworth's model of function of representations, CRM model, internal-external/abstract-concrete MER trade-off model, RCA model and distributed cognition framework); the third set of theoretical frameworks models the cognitive mechanisms involved in the processing of MERs (Dual coding theory and models informed by this view, and the model of four cognitive components). Among the RC developmental frameworks, we reviewed stage models proposed by Dreyfus (1991), Duval (2006), Goldin and Kaput (1996), Kozma and colleagues' RC characterisation, and the model of native and meta-RC by diSessa and co-workers.

In the section below, we discuss empirical studies investigating different aspects of RC, as well as the studies examining the process of learning through MERs.

## 3. Empirical investigations of learning with MERs and RC

There is a vast literature reporting empirical investigations of RC, mostly examining the learning and use of scientific and mathematical MERs, the use of MERs in science practice, and the nature of skills involved in RC. These studies are widely dispersed, and often published in discipline-specific venues. Only a few studies explicitly subscribe to one of the specific models discussed in previous sections (e.g. Aurigemma et al., 2013; Hinton & Nakhleh, 1999; Madden et al., 2011; Moore et al., 2013, etc.). Most studies only broadly relate to the major theoretical frameworks of RC. As a consequence, there is no well-articulated theoretical framework that helps integrate the disparate studies.

In this section, we bring together these disparate studies along two major themes based on the RC abilities they focus on: (1) linking MERs and translating between them, and (2) expert or student generation of MERs and their MER preferences.

Some studies discussed under each theme explicitly appeal to the classical information processing paradigm in order to explain student learning difficulties and/or expert-novice differences in relation to MERs, while some other studies implicitly assume classical information processing perspectives, but do not endorse this view explicitly. These studies focus on one or more of the following processes, usually identified with the classical information processing paradigm: working/short-term memory, long-term memory, information storage (assumes a storage module), memory/information extraction (assumes searching), translation of information (assumes that information from one code is translated into other code(s)) and fully internal representations. A third set of studies are neutral on the nature of MERs and RC. Finally, a few studies subscribe to recent cognition theories such as distributed and/or embodied cognition.

### *3.1. Linking MERs, translating and/or transforming between them*

Students find it difficult to understand the interrelations between different symbolic representations, which capture different features or aspects of worldly phenomena, and a wide range of studies have examined this difficulty. For instance, information about a chemical reaction is embedded in the symbols and numbers in the chemical equation representing that reaction. Being able to relate symbols and numbers with the dynamic reaction, by cross-linking the 'three thinking levels' (Hinton & Nakhleh, 1999), is one way to make sense of a chemical equation. Studies show that students lack a clear understanding of basic concepts such as oxidation numbers, ionic charge, atoms and atomic structure, formal rules for writing molecular formulas, as well as meaning of subscript letters, numbers and coefficients (Garforth et al., 1976; Savoy, 1988). Because of this, students face difficulties while dealing with chemical equations. In addition, students fail to associate the 'symbols and numerical answers with real objects and phenomena' when asked to explain different chemical equations (Herron & Greenbowe, 1986) using particulate drawing (Sanger, 2005). Studies that examine how students balance chemical equations (asking them to explain their balancing protocol) reveal that many students balance chemical equations algorithmically, without actually understanding the meaning of symbols and numbers (Hinton & Nakhleh, 1999; Nurrenbern & Pickering, 1987; Yarroch, 1985). The algorithmic approach to equations could indicate failure in understanding that the coefficient and subscript numerals are not just some numbers, but represent and quantify the particulate nature of matter. This is a failure in establishing the correspondence between macro-level visible reality and the periodic table, symbols and numbers, chemical formulas and reaction mechanisms.

In mathematics, many studies examining the understanding of length and area measurement show that children often struggle to see the relationship between numbers, units and space; particularly how a numerical value is related to a spatial area (Battista, 2003; Battista & Clements, 1996; Kamii & Kysh, 2006; Pande & Ramadas, 2013), even though all these different representational systems model the same concept. Santos (1996) examination of students' responses to contextualised hypothetical questions (such as 'how many tennis balls would it take to fill a classroom?') reveal that students' use of numbers, and algebraic as well as arithmetic operations, is largely algorithmic.

A related set of studies analyses the connections students and teachers make between MERs (particularly graphs, tables and pictorial representations) of mathematical functions (Çelik & Sağlam Arslan, 2012; Hitt, 1998; Knuth, 2000) by documenting which MERs were preferred by the participants over others. Knuth (2000) presented high school students with several function problems using algebraic and graphical representations, and asked them to solve each problem using either a graph or an equation, and then produce an alternative solution method using the other representation. The author found that graphical representation provided during the study was often considered irrelevant by the students. Most students prefer algebraic/symbolic representations (Acevedo Nistal, Van Dooren, Clarebout, Elen, & Verschaffel, 2010; Acevedo Nistal, Van Dooren, & Verschaffel, 2012; also shown in probability tasks by Anastasiadou & Chadjipantelis, 2008). Students did not agree on which representation would be appropriate for a problem, and found it difficult to explicitly reason using chosen representations (Acevedo Nistal et al., 2012). Learners also have a general difficulty establishing links between problem situations (word problem statements), graphs and functions and other symbolic representations (Billings & Klanderman, 2000).

Elia, Panaoura, Eracleous, and Gagatsis (2007) used different tasks that required students to explicitly talk about their definitions and understanding of the concept of functions, identify correct algebraic functions in relation to diagrams of situations and translate between multiple representations of algebraic functions. The authors report remarkable inconsistencies among students in relation to: (1) approaches to different representations of functions across tasks, and (2) definitions of functions and their ability to recognise the concept of function in different forms or problem-solving tasks. They conclude that students tend to highly compartmentalise the concepts taught to them, based on differences in the situations and representations encountered around those concepts. Students were also found to perform badly in relating situation diagrams and algebraic functions.

In chemistry, Kozma and Russell (1997) report an expert versus novice study, where they posed two tasks, a categorisation task and a transformation task, to experts (practising chemists) and novices (undergraduate students), individually. The authors wanted to know if the participants 'saw connections between different chemical visualisations corresponding to the same phenomena or if they understood something different for each type of visualisation'. The first task required participants to group a set of 14 cards, with dynamic and still images (corresponding to several chemical reactions), into meaningful groups. The representations (dynamic and static images) included videos of the experiments, animations of the molecular events, graphs, and chemical equations. Observations revealed that novices formed their meaningful groups from a small number of cards, often from the same media type (e.g. all graphs as a category, all equations as another category and so on), while experts made larger groups, composed of multiple media forms. Experts gave largely conceptual reasons for forming particular groups, while novices' reasons were often based on surface features. In the second, i.e. transformation task, participants were shown chemical equations, videos of experiments, dynamic graphs and animations of the molecular events of an experiment, one at a time. Participants were asked to transform the given representation to another form (such as drawing a graph corresponding to the given equation, selecting an animation that best corresponds to an equation, etc.). The authors found that 'experts were much better than novices at providing verbal descriptions, due to their deeper understanding of chemical principles and concepts'. Also, experts were better than novices when transformations required a constructed response, such as drawing a graph or writing a chemical equation.

Similar findings are reported by Madden et al. (2011) in their examination of RC differences between first semester and advanced level chemistry students' in the context of ideal gas problems. The problems, investigating RC level among students, required the students to provide verbal descriptions of behaviour of an ideal gas (from particulate level sketches, diagram and graphs), calculate (i.e. provide a mathematical representation of) pressure exerted by a gas and transform between these generated calculations (mathematical) and verbal descriptions, graphs and particulate level sketches. The authors used a modified version of Kozma and Russell's (2005) RC framework to analyse student performance, and found that the students with less prior experience largely exhibited algorithmic use of the ideal gas law. Their use of equation, variables and values seemed to be disconnected from other representations, unlike students with more exposure.

Ben-Zvi, Eylon, and Silberstein (1988, 1987) found that students' thinking relies primarily on perceptual/sensory information, and since the pedagogical practices while teaching symbols, equations and operations do not seek to provide perceptual-sensory assistance, these aspects of science and mathematics are not understood by students in terms of their macro- and micro-level instantiations. As a result, learners tend to concentrate more on the familiar representation(s) or algorithms in order to manage the cognitive load, and end up ignoring the relationships between concrete and abstract MERs (Johnstone, 1991; van Someren, Reimann, Boshuizen, & de Jong, 1998).

Ozogul, Johnson, Moreno, and Reisslein (2012) also focus on the load on working memory students experience while learning MERs. They examined the effects of various modes of integrating equations in circuit diagrams in the engineering domain, and found that undergrad students often failed in integrating the two kinds of representations, because of the increase in cognitive load during the instructions. The ability to establish relevance, given the information depicted through MERs, is related to the amount of information working memory can handle (Chi, Feltovich, & Glaser, 1981). Higher prior knowledge facilitates identification of the relevant/necessary features in representations, and in extraction as well as interlinking of information through these features (Chi et al., 1981; Cook, Wiebe, & Carter, 2008; Kozma & Russell, 1997; Larkin & Simon, 1987). This lowers cognitive load, as participants with higher prior knowledge can rely on their existing knowledge stored in long-term memory for information chunking.

The prior knowledge effect was demonstrated using eye tracking by Cook et al. (2008), using a study examining the way students' prior knowledge interacted with the way they interpreted macro and molecular graphics of diffusion phenomena. The authors captured the number of transitions students made between molecular-to-molecular, macro-to-molecular, molecular-to-macro and macro-to-macro representations, using eye-tracking. On average, students with low prior knowledge made more transitions than students with high prior knowledge. Students with low prior knowledge focused more on surface features of representations (Kozma & Russell, 1997). Low-prior-knowledge students needed to make frequent transitions in order to map features from one representation to the other, when trying to link them together. Similar patterns of transitions between MERs are reported by Kohl and Finkelstein (2008) in a study to understand patterns of MER use during problem-solving in electrostatics. Three groups of participants – experts, weak novices and strong novices – individually solved two different sets of problems. In one set, MERs were given to the participants. In the second set, word problems on electrostatics were given, and participants had to generate representations based on the textual description. Results showed multiple levels of competences across all the three groups, but experts (as originally designated) generally tended to successfully solve problems, making less number of to-and-fro transitions (measured as density of transitions between representations per minute) than the novices. Surprisingly, strong novices exhibited intermediate performance. Since participants with low prior knowledge are less aware of the 'subtleties of representations and the conventions for interpreting them', they may have needed more transitions to interpret the represented information (Cook et al., 2008) and relate it to information in other representations. Interestingly, researchers have found that although the domain knowledge and RC are interconnected, RC can be predicted from, but cannot guarantee, domain knowledge (Nitz, Nerdel, & Prechtl, 2012; Nitz & Tippett, 2012).

A parallel set of studies argues that visuo-spatial thinking ability is fundamental to RC, although working memory capacity is the ultimate limiting factor. Bodner and Domin (2000) examined transforming of 2D representations into 3D and the reverse, and document the difficulties students encounter with such transformations, especially in the context of organic chemistry. There is a deep relationship between students' mental rotation ability and their ability to transform 2D representations into mental 3D representations (Shubbar, 1990; Wu & Shah, 2004). Shubbar (1990) attributes students' difficulty in 2D–3D transformation to problems in either comprehending depth cues in 2D diagrams, or tracking the depth cues in molecular diagrams that depict chemical change. These simultaneous activities put tremendous cognitive load on student, and are critical to learning difficulties (Wu & Shah, 2004). In chemistry, a learner needs to perform multiple operations at multiple spatial scales: atoms and molecules, their collective behaviour and properties and reaction mechanisms need to be imagined simultaneously in a consistent manner. Similarly, understanding biological phenomena such as evolution, for instance, requires traversing different levels of spatial scales, right from DNA mutation to changes in an organism across generations.

The empirical work discussed so far largely advocates vocabulary and notions usually identified with the classical information processing theories in cognition. Building from such studies, there have been a number of attempts since the early 1990s to develop chemical visualisation/virtual manipulation software to help students develop RC. These interventions are based on the classical information processing approach to cognition, particularly Baddeley's working memory model (e.g. SMV Chem, visChem, 4M:Chem, EduChemHS, eChem, etc.), and seek mostly to *display* multiple representations simultaneously on screen, to lower the load on students' memory.

There is also a significant number of studies in RC either disregarding the concepts such as working memory and/or cognitive load, or employing alternative perspectives on cognition. For instance, Kozma et al. (2000) anchor their work in the situated cognition perspective – which proposes that knowledge of a practitioner (say a chemist) is inseparable from the natural context of that practitioner (chemistry laboratory), and is therefore best investigated within the context of that practice. The researchers observed chemists and academicians practising in laboratories, and reported that 'materialising' representations that could be perceived and manipulated, (i.e. creating and/or using MERs) helped participants operate on the otherwise imperceptible entities and processes. MERs also helped chemists discuss problems; they used visualisations and structural diagrams to describe the composition and geometry of the compounds considered for synthesis, and used diagrams and equations to think through the possible reaction mechanisms. Kozma (2003) extended this earlier study with novices (undergraduate students), and reported lack of such RC in this group.

Shifting the focus from students, Stewart (1982, 1983) argues that the origin of student difficulty in interlinking multiple representations lies in the teaching sequence of concepts. Taking examples from biology, the authors argue that teaching Mendelian genetics before cell division could be one reason why students fail to understand the connection between meiosis (micro-level explanation) and Mendelian genetics (macro level). Longden (1982), on the other hand, situates the root of the problem in the static nature of diagrams used in science classrooms.

Schnepp and Nemirovsky (2001) emphasise the role of dynamics in understanding MERs in the context of physics, and argue that the recognition of motion in distance–time, velocity–time and other equations and graphs of motion requires merging perception with imagination. They found, through observing a calculus course for 12th graders, that students have difficulties in imagining motion depicted in mathematical representations of physical phenomena. Sometimes these depictions refer to physically impossible events. For instance, a distance–time graph depicting a plane after a slope refers to an object instantaneously stopping its motion. Imagination is key to recognising the relevance of such representations to physical phenomena (Schnepp & Nemirovsky, 2001), and thus understanding the conceptual content of MERs. Thompson and Sfard (1994) argue that mere perception of a function, for instance, in its multiple representations such as a table and a graph, may not be sufficient for a student to realise the equivalence between those representations (also suggested by Kaput, 1995).

It is extremely difficult to gauge if a student understands the continuity of that function, distributed across those multiple representations.

Similarly, White and Pea (2011), during observation of students collaboratively solving a set of decryption problems using a dynamically linked multiple representation environment (Code Breaker), discovered that although students may exhibit competence relative to a specific task during problem-solving episodes, understanding that the concepts and mathematical operations are distributed across multiple representations may take numerous episodes of using multiple representations (Giere & Moffatt, 2003) as well as manoeuvring different representational tools (Hutchins, 1995a; White & Pea, 2011) collaboratively in different problem situations.

Close to the view, we advocate later in Section 5, a group of researchers argue that understanding scientific phenomena not only requires seeing the different connections between MERs, but also using those MERs and the connections between them, to build dynamic internal (mental) models that simulate the behaviour of many individual components of real-world events (Davidowitz, Chittleborough, & Murray, 2010; Grove, Cooper, & Cox, 2012; Levy & Wilensky, 2009) and effects of various parameters on such events. Difficulty in building consistent internal models of phenomena using MERs is a major problem identified among students. For instance, students are reported to have difficulties in mentally animating as well as simulating physical systems (such as flush-tank, gears; Hegarty, 2004; Schwartz & Black, 1996a, 1996b). This leads to problems in understanding and predicting system behaviour and/or answering problems.

Pande and Chandrasekharan (under review) found that expertise in a domain is accompanied by fine-tuning of the sensorimotor system, as a function of the amount of training in that domain. During an MER card sorting experiment involving the use of eye tracking, the authors discovered that experts navigated MERs in a very balanced manner in terms of eye movements. Their gaze transitions between different parts of a representation were meaning-driven and globalised, i.e. evenly distributed across the different parts of a representation (e.g. in case of an equation, the experts would specifically attempt to relate its different parts – reactants and products). Novices, on the other hand, exhibited more localised but often haphazard gaze and eye movement patterns that lacked coordination between the different regions of a representation. However, novices that performed like experts in the card-sorting task had eye movements similar to experts. The authors conclude that sensorimotor structures are tuned through interactions with MERs, and this tuning may be helping experts to pick up information from MERs more effectively.

Unlike the previously discussed computer interventions based on memory-based approaches that focus on simultaneous display, recent work informed by emerging cognition perspectives focuses on interlinking representations through dynamic *manipulable* simulations, animations and physical models. For instance, the manipulability feature in the Connected Chemistry Curriculum, based on the Netlogo 2D interface, may help students transform better between static and dynamic representations (such as equations, graphs and molecular simulations). Control-treatment group experiments, where students were asked to draw submicroscopic pictures for certain chemical systems/reactions, showed that the connected chemistry curriculum improves handling and understanding of multiple representations in chemistry, when compared to conventional text or lecture-based curricula (Stieff & McCombs, 2006; Stieff & Wilensky, 2003).

Kothiyal et al. (2014) report in detail the development and testing of a *fully manipulable* simple pendulum simulation designed to help high school students integrate MERs related to the concept of oscillation. The design principles behind this simulation are inspired by distributed and embodied cognition perspectives (e.g. external representations allow processing not possible/difficult to do in the mind, Kirsh, 2013; action patterns can activate concepts, hence actions and manipulations of the representations should be related to existing concepts, O'Malley & Soyer, 2012). Unlike the Netlogo and PhET simulations, this simulation focuses on the *enactivity*/manipulability of abstract MERs, particularly MERs such as equations/graphs, in order to give the learner maximum control over the behaviour of the system through multiple modes. The authors claim that the *enactivity* of equations and abstract

MERs is critical for understanding (implicitly as well as explicitly) the dynamic relationship between these MERs, and thus imagine the represented entity/phenomenon.

Such manipulable interfaces have often been coupled with other scaffolds (such as exercises, quizzes, activities and teacher guides; Kukkonen, Kärkkäinen, Dillon, & Keinonen, 2013; Varma & Linn, 2011) and these have been effective in improving students' representations and understanding. In organic chemistry, the activity of matching physical models to diagrams has been shown to provide (implicit) feedback to participants, leading to their improved performance during representational tasks (Padalkar & Hegarty, 2015). Computer interfaces have been explored from an assessment viewpoint in order to better characterise RC and multiple representational transformations among learners. Stieff, Hegraty, and Deslongchamps (2011) examined students' use of a multi-representational molecular mechanics animation using eye-tracking, and observed that students mainly used graphical and model representations in animations, and often ignored the equation. Based on an eye-tracking investigation of participants' MER viewing as well categorisation processes, Pande and Chandrasekharan (2014), and Pande, Shah, and Chandrasekharan (2015) concluded that the richness of gaze transitions, as well as the nature of those transitions, between different parts of a representation (and/or different representations) could be considered a good marker of MER integration.

### 3.2. Generating MERs and representational preferences

Representations generated by students, and their choices of representations (e.g. which representation would help better in a given problem situation), are considered good indicators of misconceptions (as these reflect internal representations, Chi et al., 1981). They also suggest how different representations aid student thinking (as they support reasoning during the problem-solving process, Izsák, 2011). Literature in this area documents different representational preferences among students, and suggests that students find it extremely difficult to generate MERs and use the generated MERs to reason about phenomena in systematic ways (Diezmann & English, 2001; Kamii, Kirkland, & Lewis, 2001; for detailed review, see Diezmann & English, 2001).

Many of the classic studies in science, mathematics and engineering education focus on the nature of representations participants generate during problem-solving. These generated MERs are considered markers of participants' problem representations (or internal representations/mental constructs of problem situations). Extensive work, particularly in the 1980s, investigated the way experts and novices approach physics problems, and the studies found certain key qualitative differences between the two groups, particularly in their problem representations. Chi et al.'s (1981) influential study, for instance, found that experts and novices categorise the given physics problems into different groups. The categories and explanations generated by experts had few features in common with those provided by novices. Experts sorted problems on the basis of principles, such as Law of Conservation of Energy, which could be used to solve the problems. Novices, on the other hand, exhibited limited capabilities in going beyond surface features of the problem statements/diagrams (such as literal meanings of words) while categorising problems. For instance, they put 'merry go round' and 'rotating disc' problems in the same category, as both involved rotating things. To explain these differences, Chi et al. (1981) postulated that differences in prior knowledge of the experts and novices make their problem schemata different from each other. The problem features engaged more tacit knowledge in the case of experts (Chi et al., 1981).

Interestingly, the authors found that both experts and novices used the same set of features in problem statement (and/or diagram), but the differences lay in the cues and interactions those features had with their prior knowledge and subsequent problem schemata. Participants' prior knowledge and their ability to identify patterns of meaningful information (in and using MERs) were closely related. Experts (generally assumed to possess denser domain knowledge) are more likely to extract task-relevant knowledge from a given representation or generate one to aid problem-solving (Chi et al., 1981; Larkin, McDermott, Simon, & Simon, 1980).

In Hmelo-Silver and Pfeffer's (2004) investigation of pictorial representations (and verbal responses) of models of aquatic systems generated by experts and novices, the experts were found to integrate structural–functional–behavioural information by dynamically imagining the mechanistic relationships between them; novices relied on the static structural features of the system components. Several other studies examining structure–function relationships report similar findings (e.g. Jacobson, 2001; Mathai & Ramadas, 2007; Subramaniam & Padalkar, 2009).

A related strand of research looks at how experts and novices differ in the way they use analogies to understand and explain biological phenomena (Dreyfus & Jungwirth, 1990). Student participants were asked to explain the meaning of various statements, such as – 'the nucleus controls the functioning of the cell'. Most participants used fallacious analogies, such as – 'just as brain controls the body', in their responses. Experts often used analogies from systems they understood better, but they also searched for potential mismatches in the analogies. The novices were satisfied with the criterion of familiarity with the system while choosing an analogy, and never checked the analogies for mismatches. The authors suggest that mismatches in analogies may result in inconsistencies among internal representations, difficulties in understanding MERs, and ultimately difficulties in understanding biological phenomena. Analogies are thus powerful yet risky tools in interlinking information at multiple levels of organisation (Dreyfus & Jungwirth, 1990).

Santos (1996) examined students' responses to spatial problems that required generation and use of multiple representational approaches. When students were asked to estimate the number of tennis balls needed to fill a classroom, they often attempted to translate the word problem into calculations without completely understanding the problem. They tended to use arithmetic and/or algebraic approaches to solve such problems, and had difficulties in moving from the arithmetic representation to visual estimations. Billings and Klanderman (2000), in the context of problems on motion and related topics in physics, found that students (pre-service teachers), when given graphical representations showing relation between speed and other variables (time–distance graphs), excelled at generating symbolic representations and operating on them (e.g. calculating average speed). However, the same students struggled in generating reasonable graphical representations and interpreting them while designing question sets for school exams. Further analysis of the assignments and question sets submitted by the students revealed that the students found it difficult to distinguish average speed from instantaneous speed, and even distance and speed. The slope of the line was often an area of misinterpretation and confusion.

On the other hand, students are reported to exhibit sophisticated reasoning around their choice of representations. For instance, 14-year olds, when posed with three kinds of tasks based on the design and working of a physical device called 'winch' in a study, generated many different equations, developed criteria to evaluate these equations, and finally selected some equations based on these criteria (Izsák, 2011). Interestingly, the participants developed as well as articulated their own criteria, such as 'single equation is better over multiple ones' and that 'the expression must generate positive values for distance', during the selection and evaluation of generated representations. Although students lacked coordination between their criteria while evaluating MERs generated by them, the fairly reasonable articulation of criteria hint at some competence among students in evaluating and integrating MERs.

Such seemingly pragmatic representational preference tendencies (reasoning) among students are highly context-dependent, and there may be significant individual variations (Çikla & Çakiroglu, 2006). For instance, equations are preferred during mathematical situations, whereas graphs are used when encountered with contextualised word/mathematics problems (Keller & Hirsch, 1998; Scanlon, 1998). Students feel comfortable in using only symbols in fraction problems, whereas they encounter difficulties in relating concrete models/visuals of fractions with number line, verbal and symbolic representations (Biber, 2014; Brenner, Herman, Ho, & Zimmer, 1999).

Novices have relatively unstable internal representations of problem situations than experts (Anzai, 1991; Anzai & Yokoyama, 1984); this possibly arises from the limited capacity of their working memory. In a problem-solving experiment, expert and novice participants were asked to predict the behaviour of a constrained system (Figure 8), a yo-yo made by connecting the centres of two circular discs with an axle. The system was kept on a table, in such a way that the yo-yo discs could roll, but not slide (Anzai & Yokoyama, 1984; re-described in Anzai, 1991). Participants were asked whether the yo-yo would roll

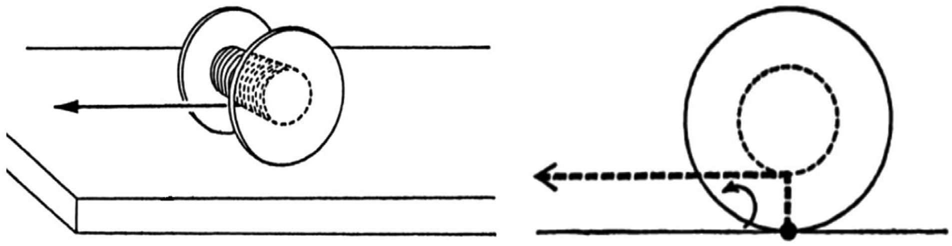

**Figure 8.** Yo-yo on a table problem (adapted from Anzai & Yokoyama, 1984). The problem diagram is on the left, and on the right is its abstract diagram.

to the left or to the right (left being the correct answer). Experts applied the lever-fulcrum principle to answer this problem correctly, while novices related the problem to real-world situations, trying to erroneously animate the yo-yo and thus performing poorly. However, changing the problem representation to abstract diagrams helped some novices to answer the yo-yo problem correctly (Anzai, 1991), possibly because of the reduction in cognitive load provided by the more directed animation of the movement of yo-yo. This result supports the suggestion from Anzai (1991) that experts and novices tend to use qualitatively different internal spatial representations to solve problems. Experts could be performing better in this generation task because producing and/or using diagrams allow computationally more efficient search for stored tacit information, as well as inference based on this information, compared to symbols and sentences (Kozma, 2003; Larkin & Simon, 1987).

A related strand of research examines ways to improve generation and integration of MERs among students, using different approaches. Cardella, Atman, and Adams (2006) investigated the role of sketching and MERs through a verbal protocol case study analysis of engineering students' representations and representational activities during a design problem-solving process. Participants had a general tendency to use the given MERs, rather than generate new ones. However, those who could generate relevant MERs, such as problem statements, sketches and calculations, exhibited 'information gathering' during the process, to progress through the task. Based on this result, the authors suggest that generation of MERs compensates for the limitations of imagery. Successful problem solvers tend to construct more accurate, complete and abstract representations (numerical/symbolic/mathematical forms) of the problems, than the unsuccessful ones. Representations generated by successful problem solvers also evolved over time in terms of their abstractness (Domin & Bodner, 2012; Sevian, Bernholt, Szteinberg, Auguste, & Pérez, 2015), suggesting addition of information from participants' prior knowledge.

Reisslein, Moreno, and Ozogul (2010) distinguish between abstract and contextualised engineering MERs, and emphasise the use of instructions that involve both these kinds. The instructions allow students to get more opportunities to interconnect abstract MERs, such as equations, with real-life situations. The study assessed learning outcomes and programme rating, using a survey based on a post-test. Three groups of participants received three different types of MER-mediated instructions: abstract (e.g. equations), contextualised (e.g. only circuit diagrams) and combined (both equations and circuit diagrams). Participants who received the combined contextualised-abstract instruction scored higher on the post-test, produced better problem representations, and rated the programme's diagrams and helpfulness higher than their counterparts.

Based on the Lesh translation model (discussed in 2.1), model eliciting activities – particularly specific and goal-directed activities that involve building a working model of phenomena using MERs – have been reported to be effective interventions in engineering education (e.g. Diefes-Dux, Moore, Zawojewski, Imbrie, & Follman, 2004; Moore et al., 2013). Moore et al. (2013) investigated how engineering students used representations and representational fluency in modelling heat exchange, and what role representations and representational fluency played in conceptual development during this activity. The students were expected to develop a model to 'predict the interface temperature and the sensation felt by human skin when touching a utensil made of a specified material at a given temperature'. Student-generated representations were grouped under the five categories provided by the Lesh translation model, viz., concrete, pictorial, symbolic, language and realistic. Model development was found to be a function of representational fluency, involving not only generation and use of MERs, but also translation across the five categories of representations, and among multiple representations belonging to the same category. Going through the process of model development also often improved representational fluency among students (Moore et al., 2013).

Kindfield (1994) demonstrated, through an empirical study, the role of diagram generation in improving working memory. Generating diagrams helped connect external representations/models, internal representations and conceptual knowledge. The study analysed students' ability to generate diagrams during meiosis (cell division) problem-solving, and the quality of the generated diagrams among participants with varying degree of formal training in genetics (meiosis – cell division). Two criteria were used to distinguish the participants: (a) number of different representations of chromosomes used to reason about meiosis, and (b) nature and timing of inclusion of different features of representations. These two criteria determine a 'knowledge-dependent representational variability' (Kindfield, 1994). Similar to RC, this concept captures the quantity and quality of variations in the use and generation of MERs. Expert problem solvers exhibited knowledge-dependent representational variability, fine-tuned their diagrams according to the nature of the task, and used them systematically during reasoning. A cyclical approach was observed in experts' problem-solving process. They first drew diagrams that offloaded their mental model, and then paused over the drawn figure, where they offloaded the computation of chromosomal configurations (essential for correct reasoning), and then draw again to externalise and check solutions, while also keeping track of the previous steps. Expert problem solvers thus exhibit better working memory skills mediated by diagrams (as the cyclical approach indicates) than novices, and these memory skills and conceptual knowledge co-evolve (Kindfield, 1994).

Anzai (1991) suggests repeated generation of MERs as an intervention to improve RC. In a repeated physics problem-solving experiment, where a student solved the same set of problems many times, and drew diagrams for each problem every time she solved that problem, new inference strategies were learned over time during the many iterations. The structure and quality of diagrams, and the relevance of the different components and elements in relation to the solution, increased dramatically over time, indicating that the students learned to make better transformations of the problem statements into sketches, diagrams and finally abstract free-body diagrams (Anzai, 1991). Izsák (2011) calls this process of repeated representation generation and problem-solving *adaptive interpretation,* which involves cycles of MER generation and self evaluation. MERs are generated first to interpret problems, then to solve problems; then MERs are generated again, and they are evaluated; this process continues until one gets a grip on the problem.

The repeated generation of sketches and MERs is believed to augment thinking and generation of ideas relevant to the process (Purcell & Gero, 1998). During engineering design, MERs such as graphics and sketches actively bring together an agent's explicit conceptual knowledge, cognitive experiences (Herbert, 1988) as well as implicit understanding of system behaviour. MERs can be easily and flexibly manipulated according to the needs of the design problem. Students who do sketching during problem-solving are more likely to formulate the problem precisely, meet relatively more problem constraints, and also produce quality design solutions, indicating that sketching helps in the overall design process. Also, different representations, such as problem statements, diagrams, equations and verbal descriptions, all serve different purposes for students, depending on the progress towards the solution (Cardella et al., 2006). The finding about sketching, particularly how it helps in meeting problem constraints, hints that sketching facilitates the dynamics of the design process, and also understanding the dynamics of the product, by providing an external memory while thinking and designing and ultimately lowering the cognitive load (Cardella et al., 2006).

Purcell and Gero's (1998) coherent and detailed review of various empirical and theoretical accounts of sketches in engineering design argues that generation of representations during the design process facilitates reinterpretation of the design itself, and this eventually leads to the emergence of new ways of *seeing* into the design. Using a design process analysis of participants from mechanical, instructional and architecture design, Goel (1995) showed that the structure of their sketches improved as the design process progresses. The designers gradually added precise details to an initially vague sketch. However, during the process, the designers often tried out different design ideas in the sketches (lateral transformations), one of which is then narrowed down and fixed as a theme to which details are added (vertical transformation). These processes of lateral and vertical transformations can be seen as a result of reinterpretation (Purcell & Gero, 1998; also designated as 'restructuring' by Cardella et al., 2006). Sketching facilitates reinterpretation by creating a perceptual space, and subsequently a conceptual space (Herbert, 1988), of many relevant ideas. The best ones are then chosen for refining.

Reinterpretation through the generation of MERs is also reported by Aurigemma et al. (2013), who observed a bioengineering researcher designing a 'Lab-on-a-chip' (LoC) device in an integrated systems biology lab. The authors, motivated by the distributed and situated cognition frameworks, report that the design activities were driven by generation of MERs, going back and forth between the MERs and the prototypes, and modifying both MERs and the prototype iteratively as a result of constant reinterpretation, in order to arrive at a well-functioning version of the prototype. The participant iteratively went through various drawings of the device, inscribed (numerals and calculations) on the drawings, generated her own drawings, imagined the structure–function relationships of the various components of the device, and tried to map them onto the requirements (often numerical in nature). She also used modelling and simulation software such as COMSOL and MATLAB to explore different design possibilities and constraints, and integrated the results with the prototype. The models output numerical data, which the student integrated with her drawings, her imagined functioning of the design and the actual test results presented by the physical prototype. Much of the cognitive activity of the student, thus, involved inferring dynamic information (in this case, the flow of liquid through the LoC device) from multiple representations (such as test results, drawings and numerical data) that were all static in nature.

### 3.4. Summary

This section reviewed empirical studies that investigated the ways in which students and experts established links between MERs and translated between them, and the patterns of MER generation as well as preferences of students and experts. We discussed how studies (from science, mathematics and engineering) reviewed under each theme assumed the classical information processing model, by highlighting classical cognition ideas such as working memory load, information storage, information extraction and translation. We also discussed studies that take a relatively neutral stance on the nature of MERs and RC, as well as studies that subscribe to recent cognition theories such as distributed and/or embodied cognition.

In the following section, we bring together the major themes revealed by the literature review presented in Sections 2 and 3.

## 4. Findings from the review

We present below four major findings from the above review of theoretical models and empirical studies of RC.

### 4.1. Ambiguous use of the term 'representation'

The term 'representation' is used often in science education literature. The review revealed that it is used in an ambiguous way, referring to internal representations, external representations or both. Some notable exceptions include: (a) problem-solving studies in physics education research, where the term 'problem representations' refers to problem solver's internal representations of the problem presented (e.g. Chi et al., 1981), and (b) several studies explicitly using the term MERs or external representations (e.g. Mammino, 2008; Nakhleh & Postek, 2008), particularly studies employing distributed cognition frameworks (e.g. Aurigemma et al., 2013; Pande & Chandrasekharan, 2014). Since the distinction between internal and external representations is not usually made, the problem of how the external and internal representations interact is rarely examined, particularly how they interact to raise or lower cognitive load, and support the imagination.

One way to think about representations in science, mathematics and engineering is to consider equations, graphs, etc. as external manifestations of experts' internal models. These external representations augment cognition by offloading memory and/or processing as well as providing novel ways of combining elements (Aurigemma et al., 2013; Kirsh, 2010). Another approach would be to consider these external media as providing starting points for the learner to develop rich internal representations and their manipulations. A third approach would be to consider the external and internal representations as being coupled, and constantly interacting with each other (Chandrasekharan, 2014). Since the literature does not make the distinction between internal and external representations, these possibilities are not examined.

### 4.2. Different nature of MERs, and RC across disciplines

Many MERs share structural commonalities across disciplines because of the intertwined nature of these disciplines. For instance, MERs in mathematics (such as equations) appear in physics, chemistry, engineering and even biology in various forms. However, there exist subtle discipline-dependent differences between these MERs and their affordances. Tables 1–6 below present a comparison of MERs in chemistry, biology, physics, mathematics and engineering across certain comparison criteria: examples of discipline-specific problems (Table 1), nature of MERs[1] (Table 2), general learning difficulties and their nature (Table 3), widely used research methods (Table 4), important theoretical frameworks (Table 5) and major interventions (Table 6). Each table has disciplines in the first column, and criterion and specific items arranged in the second column. The disciplines (first column) have been arranged sequentially across rows starting with chemistry, followed by biology, physics, mathematics and engineering on the basis of (observed) increasing complexity in the nature of MERs and the RC problem.

Table 1 captures how the disciplines differ in the nature of problems they deal with in relation to MERs, although many problems in a discipline may be interdependent and/or tightly intertwined with those in the other. For instance, relating the concept of number, mathematical operations performed on numbers and fractions to their MERs (a problem in mathematics) is fundamentally linked to balancing chemical equations.

As shown in Table 2, MERs in chemistry are more defined and constrained in nature than those in other disciplines. For instance, there are certain conventions that guide the denoting of chemical elements, compounds and other substances, writing chemical equations, plotting graphs and drawing atomic/molecular diagrams in chemistry. The periodic table is a well defined, conventionalised and compact representation of chemical elements, and their properties, and is fundamental to chemistry. Given these conventions, there is very little scope to freely generate MERs and/or alter standard chemical representations while learning/doing chemistry. There are thus limitations to the use of MERs in chemistry. MERs in biology are more diverse than those in chemistry. Biology inherits certain representational systems (essentially MERs) from chemistry, for example, chemical/biochemical equations, graphs. Phylogenetic trees in biology, like chemical equations, are MERs that are strictly conventionalised. However, macro-level biological diagrams and descriptions are quite flexible for customised usage. MERs in mathematics are highly conventionalised and rule-based. But unlike chemistry, a single concept in mathematics (such as a 'number') can be represented in multiple ways. This makes usage of MERs in mathematics more flexible for the learner or doer. Physicists employ mathematical MERs and representational systems in solving physics problems. Use of diagrams in physics is conventionalised, but the learner has enough space to generate diagrams in her own way; she can 'scribble' and represent situations under study in multiple ways and perspectives. Engineering borrows MERs from many of these disciplines, and from areas other than the core scientific domains, such as social sciences, humanities and economics. There is ample space for engineers to freely generate and play with MERs, prototypes and models. It is extremely difficult to use one kind of external representation to capture

**Table 1.** Trends in the literature on examples of problems pertaining to MERs and RC.

| Discipline | Examples of problems |
|---|---|
| Chemistry | 1. Balancing chemical equations, 2. Plotting concentration graphs, 3. Imagining reaction mechanisms, 4. Imagining chemical equilibrium, 5. Representing chemical equilibrium |
| Biology | 1. Understanding biological phenomena at multiple levels of organisation (molecular, cellular, tissue, organ, organ system, organism, community, ecology and evolution), 2. Correspondence between levels (e.g. genotype/micro-level with phenotype/macro-level) |
| Physics | 1. Producing problem-situation representations, 2. Producing mathematical models of physical phenomena/ entities* |
| Mathematics | 1. Relating concepts of number, mathematical operations, and fractions to their MERs (digits, '+', '–' signs, decimals, etc.), 2. Implicit understanding of reasoning underlying symbol systems and symbolic operations needed for working with the mathematical representation |
| Engineering | 1. Problems in designing, building devices, 2. Developing processes and systems, 3. Creating scale models, endurance-performance tests, simulations, 4. Relating engineering practice to MERs |

*Mistakes in mathematical representation are not considered, as most of the literature focuses on 'physics reasoning', and 'half-way' representations that help in the process of formalisation. Such representations are needed before the final mathematical equations.

**Table 2.** Trends in the examples of MERs and their nature.

| Discipline | MERs and their nature |
|---|---|
| Chemistry | 1. Periodic table, 2. Chemical equations, 3. Concentration-energy graphs, 4. Molecular diagrams, 5. Observable properties, 6. Animations and simulations<br>(Well defined, convention/rule-based, constrained – i.e. not very flexible, little scope for generation) |
| Biology | 1. Biochemical pathways, 2. Structures of biomolecules, 3. Phylogenetic trees, 4. Computational models of complex systems<br>(Well defined, rule-based, inclusive of but more diverse than chemistry MERs) |
| Physics | 1. Problem statements, 2. Problem situation, 3. Sketches and diagrams, 4. Mathematical equations, 5. Simulations<br>(Less defined, more customizable and less constrained, provides space for free MER generation) |
| Mathematics | 1. Digits, 2. Mathematical operations/procedures, 3. Symbols, 4. Equations, 5. Functions, 6. Charts, 7. Diagrams<br>(Well defined but more complicated MER system; allows representing a concept entirely using different representations) |
| Engineering | 1. Text, 2. Materials, 3. Inscriptions, sketches and drawings, 4. Mathematical formulae, equations and functions, 5. Prototypes and physical models<br>(highly open ended, more complicated than the previous cases, use MERs from multiple disciplines) |

**Table 3.** Similarities and differences between the disciplines in the nature of general learning difficulties.

| Discipline | Learning difficulties and their nature |
|---|---|
| Chemistry | 1. Visualisation, 2. Interconnection between MERs, 3. Representational transformation, 4. Transformation between static and dynamic MERs, 5. Conceptual integration across MERs, 6. Representation abstraction |
| Biology | 1. Understanding levels of organisation, 2. Visualisation, 3. Transition between MERs, 4. Representational transformation, 5. Conceptual integration across MERs |
| Physics | 1. visualisation, 2. Transformation between static and dynamic MERs, 3. Generation of MERs<br>4. Transformation between mathematical and real-world physical MERs, 5. Representation abstraction |
| Mathematics | 1. Transformation between spatial (e.g. area/volume) and numerical (e.g. units/numbers) MERs, 2. Generating MERs, equations, 3. Thinking in equations/functions, 4. Comprehending problem representation-situation, 6. Representation abstraction |
| Engineering | 1. Visualisation, 2. Transformation between static and dynamic MERs, 3. Generation of MERs, 4. Modelling |

every detail (feature) of the entity or phenomenon it represents. Naturally, this means MERs exist to meet this difficulty. In addition, each representation facilitates different perspectives towards entities and phenomena, as well as different affordances or action possibilities (both implicit and explicit). The exposure to multiple points of views and affordances enriches students' experiences around what is being represented, ultimately improving conceptual understanding.

MERs are complementary to each other in terms of the information they convey (Ainsworth, 1999, 2005; Kelly & Jones, 2008; Kozma & Russell, 1997; Mayer, 2005; Stieff & McCombs, 2006; Tsui & Treagust,

**Table 4.** Trends in the nature of widely employed research methods across the disciplines.

| Discipline | Research methods |
|---|---|
| Chemistry | 1. Problem posing/solving, 2. Microgenetic, 3. Ethnography, 4. Expert-novice, 5. Prior knowledge and representational competence correlation, 6. Interface testing, 7. Eye-tracking |
| Biology | 1. Prior knowledge and representational competence correlation, 2. Expert-novice, 3. Eye-tracking, 4. Interface testing |
| Physics | 1. Expert-novice, 2. MER generation and analysis, 3. Problem-solving case studies, 4. Microgenetic, 5. Design-based research, 6. Interface testing, 7. Eye-tracking |
| Mathematics | 1. Prior knowledge and representational competence correlation 2. Expert-novice, 3. Problem posing/solving, 4. MER generation |
| Engineering | 1. Ethnography, 2. Design and problem-solving case studies, 3. Design-based research, 4. Interface testing |

**Table 5.** Important theoretical frameworks of RC and learning with MERs across the disciplines.

| Discipline | Important theoretical frameworks |
|---|---|
| Chemistry | 1. Johnstone' smodel of three thinking levels and working memory, 2. Wu et al.'s MER comprehension model, 3. Abstractness of representations, 4. Distributed and situated cognition |
| Biology | 1. Multiple levels of organisation, 2. Cube model, 3. CRM mode |
| Physics | 1. Expert-novice qualitative differences (information processing model), 2. Meta-representational/native competence, 3. Abstractness of representations |
| Mathematics | 1. Lesh Translation Model, 2. Duval's levels of representational competence, 3. Abstractness of representations |
| Engineering | 1. Representational chain model, 2. Lesh translation model, 3. Situated and distributed cognition approaches |

**Table 6.** Notable trends in the major interventions across the disciplines to address the problem of RC.

| Discipline | Major interventions |
|---|---|
| Chemistry | 1. Computer visualisation tools (visChem, 4M:Chem), 2. Computer simulations, 3. Problem-based Curricula, 4. Conceptual change model, 5. Laboratory integration, 6. Sequential MER introduction |
| Biology | 1. Computer visualisation tools (evolution animations), 2. Computer simulations (Netlogo models), 3. Problem-based Curricula, 4. Laboratory integration |
| Physics | 1. Computer simulations (PhET, Netlogo), 2. Problem-context-based learning, 3. Computer visualisation, 4. Virtual laboratory |
| Mathematics | 1. Computer simulations (GeoGebra, Netlogo), 2. Problem-context-based learning, 3. Virtual/physical manipulatives |
| Engineering | 1. Computer visualisation and simulations, 2. Model eliciting activities, 3. Design and technology activities, 4. Problem-based teaching–learning, 4. STEM integration |

2003; Wilensky, 1999). On the other hand, the fact that concepts related to scientific phenomena and objects can be represented in multiple ways implies that these ideas are distributed across multiple representations. This means that the aspects of RC, particularly interconnecting information distributed across MERs, explaining the relationships between them, and mapping features of one type of representation onto those of another, are different across disciplines.

### *4.3. Integration of MERs: a general cognitive difficulty*

Despite the differences in the nature of MERs between the above disciplines, it is evident from Table 3 that the RC problem is constituted by the following learning difficulties common to all the disciplines: visualisation of or through MERs, generation of MERs to represent entities and phenomena, visualising and understanding entities and phenomena from MERs, interrelating information from MERs and transforming between MERs. Thus, difficulty in these operations underlies mastering MERs in a given discipline, and this difficulty leads to many different learning problems in that discipline. Supporting this view, in a specific knowledge domain, processing and understanding of MERs, and the ability to fluidly generate and use MERs in an integrated fashion (for conceptualisation, discovery and communication), are indicative of expertise in that domain. This suggests that RC (integration of MERs) is a general cognitive difficulty.

### 4.4. Focus on classical information processing models

Commonalities between the disciplines observed in Tables 3–6, as well as Sections 2 and 3 in the review show that most theoretical accounts of RC, as well as empirical studies and interventions across the domains, have been either explicitly or implicitly informed by the classical information processing models (Ainsworth, 1999, 2005; Johnstone, 1982; Wilensky, 1999). Chart 1 presents categories of theoretical models and empirical studies based on their subscription to major theories of cognition.

Three main assumptions can be isolated from the review in relation to these models and theories, usually also identified with the classical information processing approaches: (a) the mind extracts information from MERs, which acts as 'vehicles', or transmission media, for the information, (b) MERs and the concepts they represent are linked through some form of information 'translation' and (c) the translation is mediated mostly through mental capacities such as imagery and/or amodal symbolic forms, as well as working memory (e.g. Johnstone, 1982; Gooding, 2006; Tsui & Treagust, 2013; etc.) These assumptions, particularly limited working memory capacity as a central processing bottleneck, can be seen to influence many intervention designs. For instance, MER visualisation software used in chemistry, interactive computer simulations and virtual laboratories, are designed to address working memory limitations. Ironically, the software interventions do not seek to augment the student's working memory and processing abilities, but only help offload some of the memory and processing load to the computer screen. Possibly because of this, such interventions have not been very successful in promoting RC (De Jong & Van Joolingen, 1998; Rutten, van Joolingen, & van der Veen, 2012). Further, by focusing on the 'processor capacity' as well as the inaccessible nature of information extraction and translation processes, these models and interventions make RC appear mysterious. They do not focus on the cognitive as well as practice elements that could lead to RC development (For instance, how and why are certain interventions effective in the development of RC? What role does practice play in the RC development process, apart from enhancing working memory load abilities? What is the nature of interaction between internal and external representations? What is the role played by interactivity in simulations and other software?).

Different from such load and translation accounts, a third set of models and studies take a relatively neutral stance on the nature of MERs and RC, but these approaches do not seek to generalise, or provide detailed accounts of the cognitive processes involved in MER integration. A small set of models and studies subscribes to recent cognition theories such as distributed and/or embodied cognition. However, these fail to provide a general framework for MER integration. Without such a general account, it is difficult to develop focused new media interventions that support RC, particularly interventions that take into account the differences in MERs across different disciplines. We propose such an account in the section below.

## 5. A distributed and embodied cognition approach to RC

In our view, to develop a general cognitive account of MER integration, it is necessary to take seriously the constitutive view of external representation, and focus on the interaction between internal processes and external representations. Such an account requires moving away from the classical information processing approach to cognition, and building on newer theoretical approaches in cognitive science, particularly distributed cognition and embodied cognition. The shift to these frameworks entails rejecting the exclusive focus on cognitive load, and focusing on interactions between internal models and external representations, and the mechanisms that support this interaction.

There are two reasons why such a shift is needed. First, MERs are not just inscription devices that encode information. They are also thinking and learning devices. The process of interacting with them augments cognition, and this interaction is a central component in forming internal models of imperceptible phenomena. This means an account of the interaction process, and its role in forming internal models, is needed to understand MER integration. Such an account, outlining how interacting with external representations augment the imagination system, and cognition in general, is provided only by

### EXPLICIT SUBSCRIPTION TO INFORMATION PROCESSING THEORIES

#### MODELS

1. Johnstone's model of three thinking levels and its versions
2. Lesh translation model
3. Roth & tobin's Model of ontological gaps
4. Representational chain model
5. Dual coding theory and models based thereupon
6. Wu et al.'s model of four cognitive components (3 connections)
7. Ainsworth's framework of functions of multiple representations

#### EMPIRICAL STUDIES

Çelik & Sağlam-Arslan, 2012; Chi Feltovich & Glaser, 1981; Cook, Wiebe & Carter, 2007; Elia, Panaoura, Eracleous & Gagatsis, 2006; Garforth, Johnstone & Lazonby, 1976; Herron & Greenbowe, 1986; Hinton & Nakhleh, 1999; Hitt, 1998; Izsak, 2011; Johnstone, 1991; Kindfield, 1994; Kozma & Russell, 1997; Knuth, 2000; Larkin, McDermott, Simon & Simon, 1980; Larkin & Simon, 1987; Nurrenbern & Pickering, 1987; Ozogul, Johnson, Moreno & Reisslein, 2012; Sanger, 2005; Savoy, 1988; Shubbar, 1990; van Someren, Reimann, Boshuizen & de Jong, 1998; Wu & Shah, 2004; Yarroch, 1985.

#### MARKERS/IDENTIFIERS

All these models as well as empirical studies advocate and/or investigate working memory, cognitive load, translation of information, extraction of information, abstraction.

### IMPLICIT ASSUMPTION OF INFORMATION PROCESSING THEORIES

#### MODELS

1. Kaptejin's model of levels of organization in biology and observability
2. Dreyfus' linear stage model based on number and complexity of use of MERs
3. Goldin & Kaput's three stages of development
4. Duval's 'register' model

#### EMPIRICAL STUDIES

Ben-Zvi, Eylon, & Silberstein, 1987 & 1988; Bodner & Domin, 2000; Cardella et al., 2006; Kohl & Finkelstein, 2008;

#### MARKERS/IDENTIFIERS

(1) is based on Johnstone's model, inherits assumptions,
(2 & 3) mention abstraction (assumes translation of information in some abstract form),
(4) concerns conversion of information (as if there were a code to facilitate conversion).

All assume one or more of the following: Information translation, translation between MERs, information gathering, limitations of imagery.

### NEUTRAL

#### MODELS

1. Cube model
2. CRM model
3. Internalization-externalization model and internal-external MER trade-off model
4. Briggs & Bodner's four cognitive component model
5. Kozma's RC characterization

#### EMPIRICAL STUDIES

Anzai, 1991; Anzai & Yokohama; Battista, 2003; Battista & Clements, 1996; Billings & Klanderman, 2000; Domin and Bodner, 2012; Dreyfus & Jungwirth, 1990; Giere & Moffatt, 2003; Goel, 1995; Hmelo-Silver & Pfeffer's, 2004; Hutchins, 1995; Jacobson, 2001; Kamii & Kysh, 2006; Kaput, 1995; Kukkonen, Kärkkäinen, Dillon & Keinonen, 2013; Longden, 1982; Madden, Jones & Rahm, 2011; Mathai & Ramadas, 2007; Moore et al., 2013; Nitz, Nerdel & Prechtl, 2012; Nitz & Tippett, 2012; Pande & Ramadas, 2012; Purcell & Gero, 1998; Santos, 1996; Schnepp & Nemirovsky, 2001; Sevian et al., 2015; Stewart, 1982 & 1983; Stieff, Hegraty & Deslongchamps, 2011; Subramaniam & Padalkar, 2009; Thompson & Sfard, 1994; Varma & Linn, 2011

#### MARKERS/IDENTIFIERS

All either do not focus on how MERs are processed, or subscribe to no particular theories of MER processing.

### SUBSCRIPTION TO RECENT COGNITION PERSPECTIVES

#### MODELS

1. RCA model
2. Distributed cognition model
3. Native/meta-representational competence model

#### EMPIRICAL STUDIES

Aurigemma et al., 2013; Davidowitz, Chittleborough & Murray, 2010; Grove, Cooper & Cox, 2012; Kothiyal et al., 2014; Kozma, 2003; Kozma et al., 2000; Levy & Wilensky, 2009; Padalkar & Hegarty, 2015; Pande & Chandrasekharan, 2014; Pande and Chandrasekharan, unpublished; Pande, Shah & Chandrasekharan. 2015; Reisslein, Moreno and Ozogul, 2010; Stieff & McCombs, 2006; White & Pea, 2011;

#### MARKERS/IDENTIFIERS

(1) implicitly assumes distributed and situated approaches to cognition,
(2) is a distributed cognition model,
(3) is closer to situated cognition perspectives

**Chart 1.** Categorisation of theoretical models and empirical studies based on their subscription to general cognitive theories.

distributed cognition (DC) models, particularly recent work that combines DC with embodied cognition (Chandrasekharan, 2009, 2014, 2016; Chandrasekharan & Nersessian, 2015; Kirsh, 2010). Secondly, the central component of models in science and engineering is dynamics, and the integration of MERs requires (and happens through) understanding of dynamics, particularly the way it is captured by MERs. This means a cognitive account of MER integration requires a model of how dynamics is processed by the cognitive system, specifically how it is generated from MERs (which are mostly static), and how interactivity contributes to the understanding of dynamics. Such an account is provided only by recent embodied cognition models (Chandrasekharan, 2009, 2014; Schubotz, 2007).

Since the focus of the new account is to help individual learners integrate MERs, we will be taking an individual-focused approach to distributed cognition, which Hutchins has recently termed 'extended cognition' (Hutchins, 2014). He distinguishes it from traditional DC, which he considers a system-level theory. Similar to this scoping of the DC framework since representation (internal and external) is the focus of the account, we do not consider radical embodied cognition frameworks that reject internal representation (such as ecological psychology and dynamic systems theory), which consider sensorimotor interaction with external entities both necessary and sufficient for cognition. We accept the argument that sensorimotor interaction is necessary, and develop a framework that is based on a coupling between sensorimotor interaction and representation, using the common coding model, which is a representationalist position within embodied cognition (Chandrasekharan & Osbeck, 2010).

The account we develop thus includes the key tenets of DC (cognition as a process distributed across people and artefacts, interaction between internal and external representations), and embodied cognition (enactive and modal internal models, participatory relationship with external entities). The following are the central theoretical assumptions of the account we propose:

(1) Internal representations have a model-like structure (mental models), and they can run independent of external representations, to provide knowledge. This process provides capabilities different from the processing of external representations.
(2) Internal models have an enactive/simulative nature (Chandrasekharan, 2009; Nersessian, 2010), and they are dynamic, with a neural network like structure.
(3) Internal models interact with external ones, and they are built and extended through this interaction process. This interaction augments cognition, and it provides capabilities different from the offline processing (just internal modelling) above.
(4) The enactive nature of internal models is the key feature that enables the processing of dynamics, which is the central explanatory process in science and engineering.
(5) The integration of MERs is based on dynamics, and the mental simulation of dynamics. The motor system is the key player in the simulation of dynamics (Schubotz, 2007).
(6) The motor system is the central mediator in MER integration, as it is the major integrating system in the body (actions require integrating perception, proprioception, muscles, the vestibular system etc. in complex ways), and this integration capability is reused for other integrations, such as MER integration. This explains why the enaction/interaction features provided by new media help in understanding and learning science and engineering (and make discoveries possible using new interactive simulation systems such as Foldit), and also why integration of MERs based on static media is harder. This view also explains why activity-based classroom interventions facilitate the integration process.

Before discussing how distributed and embodied approaches to cognition could be extended to develop an account of the MER integration problem with the above features, we characterise the cognitive processes involved in a generic integration problem. This generic account can be used to examine the different cognitive frameworks, to see which provides a better understanding of this problem.

The generic case of integration of MERs in science and engineering involves the observed (or described) actual dynamic behaviour of a physical system (such as a pendulum or a falling object or a chemical process), an equation capturing this behaviour, and graphs that display the equation's output

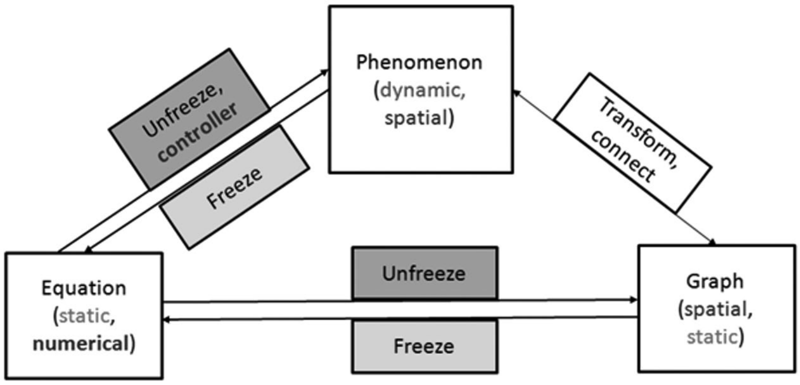


**Figure 9.** Processes involved in MER integration.

for some sets of values. The transition to the equation is often mediated by geometric structures, such as free-body diagrams and vectors, and there may be other structural representations involved, such a molecular models. Broadly though, the learner needs to develop an integrated internal representation of the three modes – the phenomenon, its equation, and the graphs. If structural representations are present, the integration process has to deal with one more level of complexity. An indicator of integration is the ability to transform smoothly between the three modes. This transformation is difficult, because it requires shifting between spatial and numerical modes (e.g. graph and equation), as well as dynamic and static modes (e.g. phenomenon and equation). Even the spatial to numerical transformation requires understanding dynamics, as the students need to understand how the values in the equation get translated into a graph, which requires thinking of various values of equations and 'movements' of the graph based on these values. Thus, to integrate the MERs, the student needs to 'unfreeze' the static representations, by generating their dynamic behaviour in imagination, and then connect these dynamics with the dynamic behaviour of the phenomenon. In the other direction, students also need to be able to 'freeze' the behaviour of real-world systems into equations (see Figure 9 below), so that limit cases and other variations can be explored and combined. This generic structure suggests a cognitive account of MER integration would need to outline:

(a) how external representations connect with imagination.
(b) how dynamic behaviour could be imagined from static external representations.

Once we have an understanding of these processes, we would be able to design interventions, particularly new media interventions that allow learners to quickly integrate MERs. Answering these two questions is not easy, as it requires bringing together complex literatures that cut across many areas of cognitive science. Answering the first question requires understanding how external representations are processed by the cognitive system. In our view, this question is best addressed within the distributed cognition (DC) framework (Hutchins, 1995a, 1995b), which was developed to study cognitive processes in complex (usually technical and scientific) task environments, particularly environments where external representations and other cognitive artefacts are used by groups of people. The DC approach was first outlined by Cole and Engeström (1993), Pea (1993), and Salomon (1993), and apart from the currently dominant model presented by Hutchins (1995a, 1995b), significant contributions to the initial framework were made by Cox (1999), Hollan, Hutchins, and Kirsh (2000), and Kirsh (2010, 1996, 2001). Most work in DC is focused on understanding how internal and external representations work together to create and help coordinate complex socio-technical systems. The primary unit of analysis in DC is a distributed socio-technical system, consisting of people working together (or individually) to accomplish a task and the artefacts they use in the process. The people and artefacts are described, respectively, as agents and nodes. Behaviour is considered to result from the interaction between external and internal representational structures.

The canonical example of external representational structures in DC is the use of speed bugs in a cockpit (Hutchins, 1995a). Speed bugs are physical tabs that can be moved over the airspeed indicator to mark critical settings for a particular flight. When landing an aircraft, pilots have to adjust the speed at which they lose altitude, based on the weight of the aircraft during landing, for that particular flight. Before the origin of the bugs, this calculation was done by pilots while doing the landing operation, using a chart and calculations in memory. With the bugs, once these markers are set between two critical speed values (based on the weight of the aircraft for a particular flight), instead of doing a numerical comparison of the current airspeed and wing configuration with critical speeds stored in memory or a chart, pilots simply glance at the dial to see where the speed-indicating needle is in relation to the bug position. This external representation allows pilots to 'read off' the current speed in relation to permissible speeds using perception. They can then calibrate their actions in response to the perceived speed difference. The speed bugs (an external artefact) thus lower the pilot's cognitive load at a critical time period (landing), by cutting down on calculations and replacing these complex cognitive operations with a perceptual operation. The setting of the speed bugs also leads to a public structure, which is shared by everyone in the cockpit. This results in the coordination of expectations and actions between the pilots. These two roles of the speed bug (lowering cognitive load and promoting coordination between pilots) are difficult to understand without considering the human and the artefact as forming a distributed cognitive system.

This account focuses on memory offloading, but it has been extended in two ways:

(1) to show how processing, particularly mental rotation, is lowered using external manipulations that serve as 'epistemic actions' (Kirsh, 2010; Kirsh & Maglio, 1994).
(2) how imagination is augmented by active manipulation, particularly in computational models (Chandrasekharan, 2009; 2014; Chandrasekharan & Nersessian, 2015; Marshall, 2007).

These studies, and other similar ones showing how external representations are used to generate action patterns (Bodemer, Ploetzner, Feuerlein, & Spada, 2004; Martin & Schwartz, 2005) suggest that the brain 'incorporates' the external representations (Chandrasekharan, 2014) as part of the imagination system. This incorporation process is considered to be driven by actions/manipulations done on the representations, and the exploration of many states of the representations. This incorporation view is different from the classical information processing view, where the information encoded in the representation is extracted by the cognitive system, and all cognitive operations are internal operations done on this extracted information. The new approach suggests that actions and manipulations on MERs lead to the MERs getting incorporated – becoming part of the cognitive system. In this view, it would be possible to improve the process of integration (of the imagination and the external representation), by restructuring the latter to support actions and manipulations, say by using new media approaches, or classroom interventions based on inquiry and activities (Lehrer & Schauble, 2006; Tytler, Prain, Hubber, & Waldrip, 2013). Such an approach to developing RC would be quite different from the approach based on cognitive load, as the incorporation approach tries to support the integration process directly using manipulations and feedback, rather than through simultaneous presentation of MERs to lower cognitive load.

The above account provides a rudimentary 'incorporation' model of how external representations connect with imagination (see Chandrasekharan, 2009; 2014 for details), and brings us to the second question: *How is dynamics generated from static representations?* Embodied cognition research argues that the brain and all cognitive processes developed for action, and the body and the motor system are therefore closely involved in most cognitive operations. Supporting this theoretical view, there is evidence that the motor system is used while generating dynamic information from static images (such as system drawings, see Hegarty, 2004) and vice versa. Everyday instances of this generation include: judging the sense of speed of a vehicle from its tyre marks (or judging tyre marks given speed), judging the sense of force from impact marks (or judging impact marks, given force), sense of movement speed from photos of action (say soccer), sense of movement derived from drawings, cartoons, sculptures, etc. Formal experimental evidence for the use of the motor system in this process comes from work on the Two-Thirds Power Law for end-point movements such as drawings and writings. The law relates the curvature of a drawing trajectory with the tangential velocity of the movement that created the drawing/writing. The human visual system deals more effectively with stimuli that follow this law than with stimuli that do not. When the curvature–velocity relationship does not comply with the power law, participants misjudge the geometric and kinematic properties of dynamic two-dimensional point-displays (Viviani & Stucchi, 1989, 1992). Also, the accuracy of visuo-manual and oculomotor 2D tracking depends on the extent to which the target's movement complies with the power law. This relation allows humans to judge the speed in which something was drawn, using curvature information, and vice versa (judge curvature given speed). This capacity is presumably what we use when we judge speed from tyre marks, and also evaluate drawings and paintings. Recent experimental evidence shows that observers simulate the drawing actions of a painter while observing paintings (Taylor, Witt, & Grimaldi, 2012). There is also evidence that object-related hand actions are evoked while processing written text (Bub & Masson, 2012).

Such predictions can also work the other way, where given a dynamic trace, we can imagine and predict the static sample that comes next. In one experiment, dynamic traces of handwriting samples were shown to participants. They were then shown some samples of written letters (such as *l* and *h*), and asked to judge which letter came next to the shown trace. Participants could identify the letter following the trace more accurately (Kandel, Orliaguet, & Viviani, 2000) when the trace followed the Two-Thirds power law, i.e. the angular momentum of writing was related to curvature in a way laid out by the law. Accuracy went down significantly for traces that did not follow this relation. Based on this and other experiments, Viviani (2002) argues that 'in formulating velocity judgements, humans have access to some implicit knowledge of the motor rule expressed by the Two-thirds Power Law'. Much of the experimental evidence in this domain is about the replication of biological movements from static images, but everyday experience (such as the tyre mark case) suggests that non-biological movements can also be replicated, and it is highly likely that this process is also based on motor system activation (Chandrasekharan, 2014; Schubotz, 2007).

This account suggests that the motor system needs to be activated to start the 'unfreezing' of MERs, to generate dynamic content using the static representation. It is possible that this activation process is difficult to do for novices, and new media interventions that allow manipulations on the MERs could help trigger this activation, thus setting the unfreezing process in motion. We are currently testing a new media system where the design provides active manipulation of all the MERs (Kothiyal et al., 2014). Note that this approach is different from the designs suggested by the cognitive load account, where manipulation of MER is not the central feature of the intervention. Also, this approach is in synergy with the 'incorporation' account provided by recent work in distributed cognition (Chandrasekharan, 2014; Chandrasekharan & Nersessian, 2015), as it suggests manipulation of the MERs as a way of promoting incorporation of the external representation with the imagination system. A related idea is that actions done on MERs with dynamic content would help improve integration, as the action system is involved in processing dynamics, and it is also the central integrating system in the body. This view provides an explanation for why interactivity provided by new media helps improve understanding and integration, and also why understanding and integration is limited with static media (Majumdar et al., 2014).

The above brief review of distributed cognition and embodied cognition approaches, and how they could together provide a general cognitive account of the MER integration problem, presents just an outline of the way these theoretical frameworks could contribute to our understanding of MER integration and RC. As of now, the two theoretical approaches only provide a way of understanding the 'unfreezing' aspect of MER integration, and how external representations could be incorporated into imagination. The frameworks do not provide a clear way of understanding the mechanisms underlying the way dynamic processes are 'frozen' into equations. The general process involved is generating a series of intermediate models (such as free body diagrams and vectors), which turn dynamics into structural features, which are then turned into equations (Karnam, Agrawal, Mishra, & Chandrasekharan, 2016).

This process is similar to the 'encapsulation' of procedures, which are considered to create concepts that embed procedural elements ('procepts'), as outlined by Tall et al. (1999).

Future work in these areas, particularly in close collaboration with science education research and new media development, may help provide a better understanding of this problem, and MER integration and RC in general. Design-based Research (Cobb, diSessa, Lehrer, & Schauble, 2003) appears to be an ideal way to bring together these disciplines to address the RC problem, as it offers a way of developing interventions that could test hypotheses about MER integration as well as cognitive processes. Our current work is focused in this direction, but we do not consider new media as stand-alone resources, to be used independent of teaching and classroom activity, as our results indicate that interaction does not automatically lead to integration – some guidance and peer discussion is required, as would be expected in the case of any cultural artefact that embeds conventions. We focus on new media because, unlike static media, they provide: (1) the possibility of making dynamics embedded in formal notations explicit and (2) action-based manipulation of this dynamics. We consider dynamics and the active manipulation of dynamics central to the integration process. Classroom activities can also be designed in such a way that they lead to interacting with the dynamics embedded in MERs. New media makes this process easier.

It is too early to make predictions about how this usage will work in practice, as there are many unknowns and possible permutations. There are groups that argue that the static MERs are outdated, as all science is moving to computational modelling, and classrooms are becoming digital. In this view, the MER integration problem is an artefact of the print media. However, there are studies showing that dynamic MERs do not by themselves solve the RC problem. We believe new media and classroom activities will support each other, but the new media will make the integration problem less formidable for both teachers and learners.

## 6. Conclusion

Integration of MERs is a critical skill in learning science, mathematics and engineering, and expertise in such integration, termed RC (RC), marks expertise in these fields. We argue that a theoretical account of RC that takes into account the constitutive character of MERs is needed, particularly to develop design guidelines for developing new embodied media interventions. As a first step to develop such an account, we reviewed the theoretical frameworks proposed for RC and related studies across the STEM domains (chemistry, biology, physics, mathematics, engineering), as well as within each domain. Addressing the results from this review, we outlined a theoretical account of RC that focuses on the interaction between internal processes and external representations, extending recent advances in distributed and embodied cognition. This proposal is preliminary, and just begins the mammoth task of developing an integrative and coherent approach to the RC problem. Particularly needed are collaborative and design-based research efforts that bring together STEM education, cognitive science and new media. Such a focused approach is required to develop theoretical and experimental frameworks that provide a better understanding of MER integration and RC. Much interesting work lies ahead.

## Note

1. Note that by MERs we mean elements of core practice. Models and simulations are not included in the review. We consider teaching–learning representations – representations used for teaching and/or learning, such as physical models and computational models for learning – different from MERs.

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## References

Abrahamson, D., & Sánchez-García, R. (2016). Learning is moving in new ways: The ecological dynamics of mathematics education. *Journal of the Learning Sciences, 25*, 203–239.

Acevedo Nistal, A., Van Dooren, W., Clarebout, G., Elen, J., & Verschaffel, L. (2010). Representational flexibility in linear-function problems: A choice/no-choice study. In L. Verschaffel, E. De Corte, J. Elen, & T. De Jong (Eds.), *Use of representations in reasoning and problem solving: Analysis and improvement* (pp. 74–93). New York, NY: Routledge.

Acevedo Nistal, A., Van Dooren, W., & Verschaffel, L. (2012). What counts as a flexible representational choice? An evaluation of students' representational choices to solve linear function problems. *Instructional Science, 40*, 999–1019.

Ainsworth, S. (1999). The functions of multiple representations. *Computers & Education, 33*, 131–152.

Ainsworth, S. (2008). The educational value of multiple-representations when learning complex scientific concepts. In J. K. Gilbert, M. Reiner, & M. Nakhleh (Eds.), *Visualisation: Theory and practice in science education* (pp. 191–208). Dordrecht, The Netherlands: Springer.

Anastasiadou, S., & Chadjipantelis, T. (2008). The role of representations in the understanding of probabilities in tertiary education. In *Eleventh International Congress on Mathematical Education*. Monterrey, Mexico.

Anzai, Y. (1991). Learning and use of representations for physics expertise. In K. A. Ericsson & J. Smith (Eds.), *Towards a general theory of expertise: Prospects and limits* (pp. 64–92). New York, NY: Cambridge University Press.

Anzai, Y., & Yokoyama, T. (1984). Internal models in physics problem solving. *Cognition and Instruction, 1*, 397–450.

Aurigemma, J., Chandrasekharan, S., Nersessian, N. J., & Newstetter, W. (2013). Turning experiments into objects: The cognitive processes involved in the design of a lab-on-a-chip device. *Journal of Engineering Education, 102*, 117–140.

Battista, M. T. (2003). Understanding students' thinking about area and volume measurement. In D. H. Clements (Ed.), *2003 Yearbook, Learning and teaching measurement* (pp. 122–142). Reston, VA: National Council of Teachers of Mathematics.

Battista, M. T., & Clements, D. H. (1996). Students' understanding of three-dimensional rectangular arrays of cubes. *Journal for Research in Mathematics Education, 27*, 258–292.

Ben-Zvi, R., Eylon, B., & Silberstein, J. (1987). Students' visualisation of a chemical reaction. *Education in Chemistry, 24*, 117–120.

Ben-Zvi, R., Eylon, B., & Silberstein, J. (1988). Theories, principles and laws. *Education in Chemistry, 25*, 89–92.

Biber, A. Ç. (2014). Mathematics teacher candidates' skills of using multiple representations for division of fractions. *Educational Research and Reviews, 9*, 237–244.

Billings, E. M., & Klanderman, D. (2000). Graphical representations of speed: Obstacles preservice K-8 teachers experience. *School Science and Mathematics, 100*, 440–450.

Bodemer, D., Ploetzner, R., Feuerlein, I., & Spada, H. (2004). The active integration of information during learning with dynamic and interactive visualisations. *Learning and Instruction, 14*, 325–341.

Bodner, G. M., & Domin, D. S. (2000). Mental models: The role of representations in problem solving in chemistry. *University Chemistry Education, 4*, 24–30.

Brenner, M. E., Herman, S., Ho, H. Z., & Zimmer, J. M. (1999). Cross-national comparison of representational competence. *Journal for Research in Mathematics Education*, 541–557.

Briggs, M., & Bodner, G. (2005). A model of molecular visualisation. In J. K. Gilbert (Ed.), *Visualisation in science education* (pp. 61–72). Dordrecht, The Netherlands: Springer.

Bub, D. N., & Masson, M. E. (2012). On the dynamics of action representations evoked by names of manipulable objects. *Journal of Experimental Psychology: General, 141*, 502.

Cardella, M. E., Atman, C. J., & Adams, R. S. (2006). Mapping between design activities and external representations for engineering student designers. *Design Studies, 27*, 5–24.

Çelik, D., & Sağlam Arslan, A. (2012). The analysis of teacher candidates' translating skills in multiple representations. *Elementary Education Online, 11*, 239–250.

Chandrasekharan, S. (2009). Building to discover: A common coding model. *Cognitive Science, 33*, 1059–1086.

Chandrasekharan, S. (2013). The cognitive science of Feynmen. *Metascience, 22*, 647–652.

Chandrasekharan, S. (2014). Becoming knowledge: Cognitive and neural mechanisms that support scientific intuition. In L. Osbeck & B. Held (Eds.), *Rational intuition: Philosophical roots, scientific investigations* (pp. 307–337). New York, NY: Cambridge University Press.

Chandrasekharan, S. (2016). Beyond telling: Where new computational media is taking model-based reasoning. In L. Mangani & C. Casadio (Eds.), *Model-based reasoning in science and technology* (pp. 471–487). Switzerland: Springer International Publishing.

Chandrasekharan, S., & Nersessian, N. J. (2015). Building cognition: The construction of computational representations for scientific discovery. *Cognitive Science, 39*, 1727–1763.

Chandrasekharan, S., & Osbeck, L. (2010). Rethinking situatedness environment structure in the time of the common code. *Theory & Psychology, 20*, 171–207.
Chi, M. T., Feltovich, P. J., & Glaser, R. (1981). Categorization and representation of physics problems by experts and novices. *Cognitive Science, 5*, 121–152.
Çikla, O. A., & Çakiroglu, E. (2006). Seventh grade students' use of multiple representations in pattern related algebra tasks. *Hacettepe Üniversitesi Eğitim Fakültesi Dergisi, 31*, 13–24.
Cobb, P., diSessa, A., Lehrer, R., & Schauble, L. (2003). Design experiments in educational research. *Educational Researcher, 32*, 9–13.
Cole, M., & Engeström, Y. (1993). A cultural historic approach to distributed cognition. In G. Salomon (Ed.), *Distributed cognitions* (pp. 1–46). New York, NY: Cambridge University Press.
Cook, M., Wiebe, E. N., & Carter, G. (2008). The influence of prior knowledge on viewing and interpreting graphics with macroscopic and molecular representations. *Science Education, 92*, 848–867.
Cox, R. (1999). Representation construction, externalised cognition and individual differences. *Learning and Instruction, 9*, 343–363.
Davidowitz, B., Chittleborough, G., & Murray, E. (2010). Student-generated submicro diagrams: A useful tool for teaching and learning chemical equations and stoichiometry. *Chemistry Education Research and Practice, 11*, 154–164.
De Jong, T., & Van Joolingen, W. R. (1998). Scientific discovery learning with computer simulations of conceptual domains. *Review of Educational Research, 68*, 179–201.
Dickes, A. C., Sengupta, P., Farris, A. V., & Basu, S. (2016). Development of mechanistic reasoning and multilevel explanations of ecology in third grade using agent-based models. *Science Education, 100*, 734–776.
Diefes-Dux, H. A., Moore, T., Zawojewski, J., Imbrie, P. K., & Follman, D. (2004). A framework for posing open-ended engineering problems: Model-eliciting activities. In *34th Annual Frontiers in Education (FIE)* (pp. F1A-3). Savannah, GA: IEEE.
Diezmann, C., & English, L. (2001). *The roles of representation in school mathematics: 2001 YearBook* (pp. 1–23). Virginia, VA: NCTM.
diSessa, A. A. (2004). Metarepresentation: Native competence and targets for instruction. *Cognition and Instruction, 22*, 293–331.
diSessa, A. A., Hammer, D., Sherin, B., & Kolpakowski, T. (1991). Inventing graphing: Meta-representational expertise in children. *The Journal of Mathematical Behavior, 10*, 117–160.
diSessa, A. A., & Sherin, B. L. (2000). Meta-representation: An introduction. *The Journal of Mathematical Behavior, 19*, 385–398.
Domin, D., & Bodner, G. (2012). Using students' representations constructed during problem solving to infer conceptual understanding. *Journal of Chemical Education, 89*, 837–843.
Dreyfus, T. (1991). Advanced mathematical thinking processes. In D. Tall (Ed.), *Advanced mathematical thinking* (pp. 25–41). Dordrecht, The Netherlands: Kluwer.
Dreyfus, A., & Jungwirth, E. (1990). Macro and micro about the living cell: Which explains what. In P. L. Lijnse, P. Licht, W. de Vos, & A. J. Vaarlo (Eds.), *Relating macroscopic phenomena to microscopic particles* (pp. 108–118). Utrecht, The Netherlands: CD-Press.
Duval, R. (2006). A cognitive analysis of problems of comprehension in a learning of mathematics. *Educational Studies in Mathematics, 61*, 103–131.
Elia, I., Panaoura, A., Eracleous, A., & Gagatsis, A. (2007). Relations between secondary pupils' conceptions about functions and problem solving in different representations. *International Journal of Science and Mathematics Education, 5*, 533–556.
Garforth, F. M., Johnstone, A. H., & Lazonby, J. N. (1976). Ionic equations: Difficulties in understanding and use. *Education in Chemistry, 13*, 72–75.
Giere, R. N., & Moffatt, B. (2003). Distributed cognition: Where the cognitive and the social merge. *Social Studies of Science, 33*, 301–310.
Gilbert, J. K. (2005). Visualisation: A metacognitive skill in science and science education. In J. K. Gilbert (Ed.), *Visualization in Science Education* (pp. 9–27). The Netherlands: Springer.
Goel, V. (1995). *Sketches of thought*. Cambridge: MIT Press.
Goldin, G. A., & Kaput, J. J. (1996). A joint perspective on the idea of representation in learning and doing mathematics. *Theories of mathematical learning,* 397–430.
Gooding, D. C. (2006). From phenomenology to field theory: Faraday's visual reasoning. *Perspectives on Science, 14*, 40–65.
Grove, N. P., Cooper, M. M., & Cox, E. L. (2012). Does mechanistic thinking improve student success in organic chemistry? *Journal of Chemical Education, 89*, 850–853.
Hegarty, M. (2004). Mechanical reasoning by mental simulation. *Trends in Cognitive Sciences, 8*, 280–285.
Herbert, D. M. (1988). Study drawings in architectural design: Their properties as a graphic medium. *Journal of Architectural Education, 41*, 26–38.
Herron, J. D., & Greenbowe, T. J. (1986). What can we do about Sue: A case study of competence. *Journal of Chemical Education, 63*, 528–531.
Hinton, M. E., & Nakhleh, M. B. (1999). Students' microscopic, macroscopic, and symbolic representations of chemical reactions. *The Chemical Educator, 4*, 158–167.
Hitt, F. (1998). Difficulties in the articulation of different representations linked to the concept of function. *The Journal of Mathematical Behavior, 17*, 123–134.

Hmelo-Silver, C. E., & Pfeffer, M. G. (2004). Comparing expert and novice understanding of a complex system from the perspective of structures, behaviors, and functions. *Cognitive Science, 1*, 127–138.
Hollan, J., Hutchins, E., & Kirsh, D. (2000). Distributed cognition: Toward a new foundation for human-computer interaction research. *ACM Transactions on Computer-Human Interaction, 7*, 174–196.
Hutchins, E. (1995a). How a cockpit remembers its speeds. *Cognitive Science, 19*, 265–288.
Hutchins, E. (1995b). *Cognition in the wild* (Vol. 262082314). Cambridge, MA: MIT press.
Hutchins, E. (2014). The cultural ecosystem of human cognition. *Philosophical Psychology, 27*, 34–49.
Izsák, A. (2011). Representational competence and algebraic modeling. In E. Knuth (Ed.), *Early algebraization* (pp. 239–258). Berlin, Germany: Springer.
Jacobson, M. J. (2001). Problem solving, cognition, and complex systems: Differences between experts and novices. *Complexity, 6*, 41–49.
Jensen, W. B. (1998). Logic, history, and the chemistry textbook: I. Does chemistry have a logical structure? *Journal of Chemical Education, 75*, 679–687.
Johnstone, A. H. (1982). Macro and microchemistry. *School Science Review, 64*, 377–379.
Johnstone, A. H. (1991). Why is science difficult to learn? Things are seldom what they seem. *Journal of Computer Assisted Learning, 7*, 75–83.
Johnstone, A. H. (2000). Teaching of chemistry: Logical or psychological? *Chemistry Education: Research and Practice in Europe, 1*, 9–15.
Johri, A., Roth, W. M., & Olds, B. M. (2013). The role of representations in engineering practices: Taking a turn towards inscriptions. *Journal of Engineering Education, 102*, 2–19.
Kamii, C., Kirkland, L., & Lewis, B. A. (2001). Representation and abstraction in young children's numerical reasoning. *The roles of representation in school mathematics, 2001*, 24–34.
Kamii, C., & Kysh, J. (2006). The difficulty of "length × width": Is a square the unit of measurement? *The Journal of Mathematical Behavior, 25*, 105–115.
Kandel, S., Orliaguet, J. P., & Viviani, P. (2000). Perceptual anticipation in handwriting: The role of implicit motor competence. *Perception & Psychophysics, 62*, 706–716.
Kapteijn, M. (1990). The functions of organizational levels in biology for describing and planning biology education. In P. L. Lijnse, R. Licht, W. de Vos, & A. J. Vaarlo (Eds.), *Relating macroscopic phenomena to microscopic particles* (pp. 139–150). Utrecht: CD-Press.
Kaput, J. J. (1995). *A research base supporting long term algebra reform?* In D. T. Owens, M. K. Reed, & G. M. Milsaps (Eds.), *Proceedings of the 7th PME-NA Annual Meeting* (Vol. 1, pp. 71–94), Columbus, OH.
Karnam, D., Agrawal, H., Mishra, D., & Chandrasekharan, S. (2016). Interactive vectors for model-based reasoning. To appear in the *Proceedings of the 24th International Conference on Computers in Education*, Mumbai: Asia-Pacific Society for Computers in Education.
Keller, B. A., & Hirsch, C. R. (1998). Student preferences for representations of functions. *International Journal of Mathematical Education in Science and Technology, 29*, 1–17.
Kelly, R. M., & Jones, L. L. (2008). Investigating students' ability to transfer ideas learned from molecular animations of the dissolution process. *Journal of Chemical Education, 85*, 303–309.
Kieran, C. (1981). Concepts associated with the equality symbol. *Educational Studies in Mathematics, 12*, 317–326.
Kindfield, A. C. (1994). Biology diagrams: Tools to think with. *Journal of the Learning Sciences, 3*, 1–36.
Kirsh, D. (1996) Adapting the Environment instead of oneself. *Adaptive Behavior, 4*:415–452. Cambridge: MIT Press.
Kirsh, D. (2001). The context of work. *Human–Computer Interaction, 16*, 305–322.
Kirsh, D. (2010). Thinking with external representations. *AI and Society, 25*, 441–454. London: Springer.
Kirsh, D. (2013). Embodied cognition and the magical future of interaction design. In P. Marshall, A. N. Antle, E. V. D. Hoven, & Y. Rogers (Eds.), *The theory and practice of embodied interaction in HCI and interaction design [Special issue]. ACM Transactions on Computer–Human Interaction (TOCHI), 20*, 1–30.
Kirsh, D., & Maglio, P. (1994). On distinguishing epistemic from pragmatic action. *Cognitive Science, 18*, 513–549.
Knuth, E. J. (2000). Student understanding of the Cartesian connection: An exploratory study. *Journal for Research in Mathematics Education, 31*, 500–508.
Koedinger, K. R., Alibali, M. W., & Nathan, M. J. (2008). Trade-offs between grounded and abstract representations: Evidence from algebra problem solving. *Cognitive Science: A Multidisciplinary Journal, 32*, 366–397.
Kohl, P. B., & Finkelstein, N. D. (2008). Patterns of multiple representation use by experts and novices during physics problem solving. *Physical Review Special Topics-Physics Education Research, 4* (1), 1–13, 010111.
Kothiyal, A., Majumdar, R., Pande, P., Agarwal, H., Ranka, A., & Chandrasekharan, S. (2014) How does representational competence develop? Explorations using a fully controllable interface and eye-tracking. In C.-C. Liu, Y. T. Wu, T. Supnithi, T. Kojiri, H. Ogata, S. C. Kong, & A. Kashihara (Eds.), *Proceedings of the 22nd International Conference on Computers in Education* (pp. 738–743). Nara, Japan: Asia-Pacific Society for Computers in Education.
Kozma, R. (2003). The material features of multiple representations and their cognitive and social affordances for science understanding. *Learning and Instruction, 13*, 205–226.
Kozma, R., Chin, E., Russell, J., & Marx, N. (2000). The Roles of Representations and Tools in the Chemistry Laboratory and Their Implications for Chemistry Learning. *Journal of the Learning Sciences, 9*, 105–143.

Kozma, R. B., & Russell, J. (1997). Multimedia and understanding: Expert and novice responses to different representations of chemical phenomena. *Journal of Research in Science Teaching, 34*, 949–968.
Kozma, R., & Russell, J. (2005). Students becoming chemists: Developing representational competence. In J. K. Gilbert (Ed.), *Visualisation in science education* (pp. 121–145). The Netherlands: Springer.
Kukkonen, J. E., Kärkkäinen, S., Dillon, P., & Keinonen, T. (2013). The effects of Scaffolded simulation-based inquiry learning on fifth-graders' representations of the greenhouse effect. *International Journal of Science Education, 36*, 406–442.
Landy, D., Allen, C., & Zednik, C. (2014). A perceptual account of symbolic reasoning. *Frontiers in Psychology, 5*, 1–10.
Larkin, J., McDermott, J., Simon, D. P., & Simon, H. A. (1980). Expert and novice performance in solving physics problems. *Science, 208*, 1335–1342.
Larkin, J. H., & Simon, H. A. (1987). Why a diagram is (sometimes) worth ten thousand words. *Cognitive Science, 11*, 65–100.
Lehrer, R., & Schauble, L. (2006). Cultivating model-based reasoning in science education. In K. Sawyer (Ed.), *The Cambridge handbook of the learning sciences* (pp. 371–388). New York, NY: Cambridge University Press.
Leinhardt, G., Zaslavsky, O., & Stein, M. K. (1990). Functions, graphs, and graphing: Tasks, learning, and teaching. *Review of Educational Research, 60*, 1–64.
Lesh, R., Post, T., & Behr, M. (1987). Representations and translations among representations in mathematics learning and problem solving. In C. Janvier (Ed.), *Problems of representations in the teaching and learning of mathematics* (pp. 33–40). Hillsdale, NJ: Lawrence Erlbaum.
Levy, S. T., & Wilensky, U. (2009). Crossing levels and representations: The connected chemistry (CC1) curriculum. *Journal of Science Education and Technology, 18*, 224–242.
Longden, B. (1982). Genetics – Are there inherent learning difficulties? *Journal of Biological Education, 16*, 135–140.
Madden, S. P., Jones, L. L., & Rahm, J. (2011). The role of multiple representations in the understanding of ideal gas problems. *Chemistry Education Research & Practice, 12*, 283–293.
Majumdar, R., Kothiyal, A., Ranka, A., Pande, P., Murthy, S., Agarwal, H., & Chandrasekharan, S. (2014, December). The enactive equation: Exploring how multiple external representations are integrated, using a fully controllable interface and eye-tracking. In *IEEE Sixth International Conference on Technology for Education (T4E)* (pp. 233–240). Kerala: IEEE.
Mammino, L. (2008). Teaching chemistry with and without external representations in professional environments with limited resources. In J. K. Gilbert, M. Reiner, & M. Nakhleh (Eds.), *Visualisation: Theory and practice in science education* (pp. 155–185). The Netherlands: Springer.
Marshall, P. (2007). Do tangible interfaces enhance learning? In *Proceedings of the 1st International Conference on Tangible and Embedded Interaction* (pp. 163–170). Baton Rouge, LA: ACM.
Martin, T., & Schwartz, D. L. (2005). Physically distributed learning: Adapting and reinterpreting physical environments in the development of fraction concepts. *Cognitive Science, 29*, 587–625.
Mathai, S., & Ramadas, J. (2007). *Visualising structure and function of the digestive system*. A poster presented at the Gordon Research Conference on visualisation in Science and Education, Bryant University, Rhode Island.
Mayer, R. E. (2005). Principles for reducing extraneous processing in multimedia learning: Coherence, signaling, redundancy, spatial contiguity, and temporal contiguity principles. In R. E. Mayer (Ed.), *The Cambridge handbook of multimedia learning* (pp. 183–200). New York, NY: Cambridge University Press.
McCracken, W. M., & Newstetter, W. C. (2001). Text to diagram to symbol: Representational transformations in problem-solving. In *31st Annual Frontiers in Education Conference* (Vol. 2, pp. F2G–13). Reno, NV.
Moore, T. J., Miller, R. L., Lesh, R. A., Stohlmann, M. S., & Kim, Y. R. (2013). Modeling in engineering: The role of representational fluency in students' conceptual understanding. *Journal of Engineering Education, 102*, 141–178.
Nakhleh, M. B., & Krajcik, J. S. (1994). Influence of levels of information as presented by different technologies on students' understanding of acid, base, and pH concepts. *Journal of Research in Science Teaching, 31*, 1077–1096.
Nakhleh, M. B., & Postek, B. (2008). Learning chemistry using multiple external representations. In J. K. Gilbert et al. (Eds.), *Visualisation: Theory and practice in science education* (pp. 209–231). The Netherlands: Spinger.
Nersessian, N. J. (2010). *Creating scientific concepts*. Cambridge: MIT press.
Nitz, S., Nerdel, C., & Prechtl, H. (2012). Modeling the relationship between representational competence and domain knowledge. In E. de Vries & K. Scheiter (Eds.), *Proceedings of EARLI SIG 2 Meeting, Staging knowledge and experience: How to take an advantage of representational technologies in education and training?* (pp. 160–162). Grenoble, France.
Nitz, S., & Tippett, C. D. (2012). Measuring representational competence in science. In E. de Vries & K. Scheiter (Eds.), *Proceedings of EARLI SIG 2 Meeting, Staging knowledge and experience: How to take an advantage of representational technologies in education and training?* (pp. 163–165), Grenoble, France.
Nurrenbern, S. C., & Pickering, M. (1987). Concept learning versus problem solving: Is there a difference? *Journal of Chemical Education, 64*, 508–510.
O'Malley, M. A., & Soyer, O. S. (2012). The roles of integration in molecular systems biology. *Studies in History and Philosophy of Science Part C: Studies in History and Philosophy of Biological and Biomedical Sciences, 43*, 58–68.
Ozogul, G., Johnson, A. M., Moreno, R., & Reisslein, M. (2012). technological literacy learning with cumulative and stepwise integration of equations into electrical circuit diagrams. *IEEE Transactions on Education, 55*, 480–487.
Padalkar, S., & Hegarty, M. (2015). Models as feedback: Developing representational competence in chemistry. *Journal of Educational Psychology, 107*, 451–467.
Paivio, A. (1991). Dual coding theory: Retrospect and current status. *Canadian Journal of Psychology, 45*, 255–287.

Pande, P., & Chandrasekharan, S. (2014). Categorization of multiple external representations by chemistry undergrads: An eye-tracking study, In Y.-J. Lee, N. T.-L. Lim, K. S. Tan, H. E. Chu, P. Y. Lim, Y. H. Lim, & I. Tan (Eds.), *Proceedings of the International Science Education Conference 2014* (pp. 1393–1415). Singapore: NIE.
Pande, P., & Chandrasekharan, S. (under review). *Expertise in chemistry is marked by sensorimotor tuning that helps pickup information quickly from external representations*.
Pande, P., & Ramadas, J. (2013). Students' measurement experiences and responses to length comparison, seriation and proportioning tasks. In G. Nagarjuna, A. Jamakhandi, & S. Ebie (Eds.), *Proceedings of epiSTEME-5* (pp. 145–151). Mumbai: HBCSE, TIFR.
Pande, P., Shah, P., & Chandrasekharan, S. (2015). How do experts and novices navigate chemistry representations – An eye tracking investigation. In S. Chandrasekharan, S. Murthy, G. Banarjee, & A. Muralidhar (Eds.), *Proceedings of EPISTEME-6* (pp. 102–109). Mumbai: HBCSE-TIFR.
Pape, S. J., & Tchoshanov, M. A. (2001). The role of representation(s) in developing mathematical understanding. *Theory Into Practice, 40*, 118–127.
Pea, R. D. (1993). Practices of distributed intelligence and designs for education. In G. Salomon (Ed.), *Distributed cognitions Psychological and Educational Considerations* (pp. 47–87). New York, NY: Cambridge University Press.
Prain, V., & Tytler, R. (2012). Learning through constructing representations in science: A framework of representational construction affordances. *International Journal of Science Education, 34*, 2751–2773.
Purcell, A. T., & Gero, J. S. (1998). Drawings and the design process. *Design Studies, 19*, 389–430.
Reisslein, M., Moreno, R., & Ozogul, G. (2010). Pre-college electrical engineering instruction: The impact of abstract vs. contextualized representation and practice on learning. *Journal of Engineering Education, 99*, 225–235.
Roth, W. M., & Tobin, K. (1997). Cascades of inscriptions and the re-presentation of nature: How numbers, tables, graphs, and money come to re-present a rolling ball. *International Journal of Science Education, 19*, 1075–1091.
Rutten, N., van Joolingen, W. R., & van der Veen, J. T. (2012). The learning effects of computer simulations in science education. *Computers & Education, 58*, 136–153.
Salomon, G. (Ed.). (1993). *Distributed cognitions: Psychological and educational considerations* (Vol. 11). Cambridge: Cambridge University Press.
Sanger, M. J. (2005). Evaluating students' conceptual understanding of balanced equations and stoichiometric ratios using a particulate drawing. *Journal of Chemical Education, 82*, 131–134.
Santos, M. (1996). An exploration of strategies used by students to solve problems with multiple ways of solution. *The Journal of Mathematical Behavior, 15*, 263–284.
Savoy, L. G. (1988). Balancing chemical equations. *School Science Review, 69*, 713–720.
Scanlon, E. (1998). How beginning students use graphs of motion. In M. van Someren, P. Reimann, H. P. A. Boshuizen, T. de Jong (Eds.), *Learning with multiple representations* (pp. 67–86). Oxford: Elsevier.
Schnepp, M., & Nemirovsky, R. (2001). Constructing a foundation for the fundamental theorem of calculus. A. A. Cuoco & F. R. Curcio (Eds.), *The roles of representation in school mathematics (Yearbook of the National Council Teachers of Mathematics)* (pp. 90–102). Reston, VA: NCTM.
Schnotz, W. (2002). Towards an integrated view of learning from text and visual displays. *Educational Psychology Review, 14*, 101–120.
Schönborn, K. J., & Anderson, T. R. (2009). A model of factors determining students' ability to interpret external representations in biochemistry. *International Journal of Science Education, 31*, 193–232.
Schubotz, R. I. (2007). Prediction of external events with our motor system: Towards a new framework. *Trends in Cognitive Sciences, 11*, 211–218.
Schwartz, D. L., & Black, J. B. (1996a). Analog imagery in mental model reasoning: Depictive models. *Cognitive Psychology, 30*, 154–219.
Schwartz, D. L., & Black, J. B. (1996b). Shuttling between depictive models and abstract rules: Induction and fallback. *Cognitive Science, 20*, 457–497.
Sevian, H., Bernholt, S., Szteinberg, G. A., Auguste, S., & Pérez, L. C. (2015). Use of representation mapping to capture abstraction in problem solving in different courses in chemistry. *Chemistry Education Research and Practice, 16*, 429–446.
Shubbar, K. E. (1990). Learning the visualisation of rotations in diagrams of three dimensional structures. *Research in Science & Technological Education, 8*, 145–154.
Stewart, J. H. (1982). Difficulties experienced by high school students when learning basic Mendelian genetics. *The American Biology Teacher, 44*, 80–89.
Stewart, J. (1983). Student problem solving in high school genetics. *Science Education, 67*, 523–540.
Stieff, M., Hegarty, M., & Deslongchamps, G. (2011). Identifying representational competence with multi-representational displays. *Cognition and Instruction, 29*, 123–145.
Stieff, M., & McCombs, M. (2006). Increasing representational fluency with visualisation tools. In S. Barab, K. E. Hay, & D. T. Hickey (Eds.), *Proceedings of the 7th international conference on Learning sciences (ICLS)* (Vol. 1, pp. 730–736). Mahwah, NJ: Erlbaum.
Stieff, M., & Wilensky, U. (2003). Connected chemistry – Incorporating interactive simulations into the chemistry classroom. *Journal of Science Education and Technology, 12*, 285–302.

Subramaniam, K., & Padalkar, S. (2009). Visualisation and reasoning in explaining the phases of the moon. *International Journal of Science Education, 31*, 395–417.

Sweller, J., & Chandler, P. (1991). Evidence for cognitive load theory. *Cognition and Instruction, 8*, 351–362.

Tall, D., Thomas, M., Davis, G., Gray, E., & Simpson, A. (1999). What is the object of the encapsulation of a process? *The Journal of Mathematical Behavior, 18*, 223–241.

Taylor, E. T., Witt, J. K., & Grimaldi, P. J. (2012). Uncovering the connection between artist and audience: Viewing painted brushstrokes evokes corresponding action representations in the observer. *Cognition, 125*, 26–36.

Thompson, P. W., & Sfard, A. (1994). Problems of reification: Representations and mathematical objects. In *Proceedings of the annual meeting of the International Group for the Psychology of Mathematics Education – North America, plenary sessions* (Vol. 1, pp. 1–32), Baton Rouge, LA.

Tsui, C. Y., & Treagust, D. F. (2003). Genetics reasoning with multiple external representations. *Research in Science Education, 33*, 111–135.

Tsui, C. Y., & Treagust, D. F. (2013). Introduction to multiple representations: Their importance in biology and biological education. In C. Y. Tsui & D. F. Treagust (Eds.), *Multiple representations in biological education* (pp. 3–18). The Netherlands: Springer.

Tytler, R., & Prain, V. (2010). A framework for re-thinking learning in science from recent cognitive science perspectives. *International Journal of Science Education, 32*, 2055–2078.

Tytler, R., Prain, V., Hubber, P., & Haslam, F. (2013). Reasoning in science through representation. In *Constructing representations to learn in science* (pp. 83–107). Rotterdam, The Netherlands: Sense Publishers.

Tytler, R., Prain, V., Hubber, P., & Waldrip, B. (Eds.). (2013). *Constructing representations to learn in science*. Rotterdam, The Netherlands: Sense Publishers.

van Someren, M. W., Reimann, P., Boshuizen, H., & de Jong, T. (1998). Learning with multiple representations. Advances in learning and instruction series. New York, NY: Elsevier Science.

Varma, K., & Linn, M. (2011). Using interactive technology to support students' understanding of the greenhouse effect and global warming. *Journal of Science Education and Technology, 21*, 1–12.

Viviani, P. (2002). *Motor competence in the perception of dynamic events: A tutorial*. In *Common mechanisms in perception and action: Attention and performance* (Vol. XIX, pp. 406–442).

Viviani, P., & Stucchi, N. (1989). The effect of movement velocity on form perception: Geometric illusions in dynamic displays. *Perception & Psychophysics, 46*, 266–274.

Viviani, P., & Stucchi, N. (1992). Biological movements look uniform: Evidence of motor-perceptual interactions. *Journal of Experimental Psychology: Human Perception and Performance, 18*, 603–623.

White, T., & Pea, R. (2011). Distributed by design: On the promises and pitfalls of collaborative learning with multiple representations. *Journal of the Learning Sciences, 20*, 489–547.

Wilensky, U. (1999). GasLab: An extensible modeling toolkit for connecting micro-and macro-properties of gases. In N. Roberts, W. Feurzeig, & B. Hunter (Eds.), *Computer modeling and simulation in science education* (pp. 151–178). Berlin: Springer-Verlag.

Won, M., Yoon, H., & Treagust, D. F. (2014). Students' learning strategies with multiple representations: Explanations of the human breathing mechanism. *Science Education, 98*, 840–866.

Wu, H. K., Krajcik, J. S., & Soloway, E. (2001). Promoting understanding of chemical representations: Students' use of a visualization tool in the classroom. *Journal of Research in Science Teaching, 38*, 821–842.

Wu, H. K., & Shah, P. (2004). Exploring visuospatial thinking in chemistry learning. *Science Education, 88*, 465–492.

Yarroch, W. L. (1985). Student understanding of chemical equation balancing. *Journal of Research in Science Teaching, 22*, 449–459.